DYNAMIC MODELING AND TRAJECTORY TRACKING CONTROL OF HEXACOPTER USING SLIDING MODE CONTROLLER

A MASTER'S THESIS

BY

SABA MULUALEM WALLE

DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

ADDIS ABABA SCIENCE AND TECHNOLOGY UNIVERSITY

JANUARY 2019

DYNAMIC MODELING AND TRAJECTORY TRACKING CONTROL OF HEXACOPTER USING SLIDING MODE CONTROLLER

BY

SABA MULUALEM WALLE

A Thesis Submitted to

The Department of Electrical and Computer Engineering for the Partial Fulfillment of the Requirements for the Degree of Master of Science in Electrical and Computer Engineering

(Control and Instrumentation Engineering)

to

DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

ADDIS ABABA SCIENCE AND TECHNOLOGY UNIVERSITY

JANUARY 2019

Declaration

I, the undersigned, declare that this thesis work entitled "**Dynamic Modeling and Trajectory Tracking Control of Hexacopter Using Sliding Mode Controller"** was developed and written by myself with the help of my advisor. The sources have not been used without the declaration in the work. The Master thesis contained herein is my own except where explicitly stated otherwise in the text and this work has not been submitted in the similar version to achieve any other academic degree or processional qualification.

Name:	Signature, Date:
Saba Mulualem Walle	________________

Witnessed by:

Name of student advisor;	Signature, Date:
Prof. Venkata Mayol L.N.K	________________
Name of student co-advisor	Signature, Date:
Mr. Beza Nekatibeb	____________

Certificate

This is to certify that the thesis prepared by **Ms. Saba Mulualem Walle** entitled "**Dynamic Modeling and Trajectory Tracking Control of Hexacopter Using Sliding Mode Controller".** In addition, submitted in the fulfillment of the requirements for the Degree of Master of Science complies with the regulations of the University and meets the accepted standards with respect to originality and quality.

Singed by Examining Board:

External Examiner:	Signature,	Date:
Dr. Fisseha	______________	___________
Internal Examiner:	Signature,	Date:
Mr. Mulugeta Debebe	______________	___________
Chairperson:	Signature,	Date:
Dr. Asrat Mulatu	______________	___________
DGC Chairperson:	Signature,	Date:
_____________________	______________	___________
Collage Dean/Associate Dean for GP	Signature,	Date:
_____________________	______________	___________

Abstract

This thesis presents dynamic modelling and design of controller for improving trajectory tracking of Hexacopter. Hexacopter is currently a popular unmanned aerial vehicle (UAV); it is used in versatile activities like security, aerial filming, fire-fighting, pesticide spraying and delivery of goods.

In this work, the nonlinear dynamic modeling of Hexacopter is formulated by using Newton Euler modeling equations. Hexacopter is a six degree of freedom aerial robot in which it has three translational (x, y and z) and three rotational (ϕ, θ and ψ) components. It has a highly nonlinear, unstable and coupled dynamics. It is an under actuated system in which there are four control inputs and six outputs. After modeling the dynamics of the system sliding mode controller (SMC) is designed to control the motion in six-coordinates of Hexacopter in space; implemented for stabilization of internal dynamics and external dynamics. Besides, the reference trajectory is generated and track this trajectory by implementing SMC controller with minimum possible error.

Finally, the thesis presents simulation results for controller action and trajectory tracking by employing MATLAB Simulink. The results shows that the controller follows the reference, in all the coordinates, with in an average of 0.5 seconds after simulation starts.

Keywords: *Hexacopter, Unmanned Aerial Vehicle (UAV), Sliding Mode Control (SMC)*

Acknowledgments

First, I would like to thank Addis Ababa Science and Technology University as well as Electrical and Mechanical Engineering College for giving me the opportunity to undertake this Master program in Control and Instrumentation Engineering stream.

It is a privilege and honor to express my sincere gratitude to my advisor Prof. Venkata Mayol L.N.K for all his kind help, guidance, suggestions and support through the whole work of this research paper. I would also like to express my sincere thanks to my co-advisor Mr. Beza Nekatibebe for all his guidance and support in completion of this thesis. I would also like to thank my classmate Mr. Biruk Tadesse for his assistance and encouragement from the beginning up to the end of this work.

Last, but not list, I give special thanks to all Electrical and Computer Engineering Department administrative and teaching staffs at Addis Ababa Science and Technology University.

Table of Contents

List of Tables

List of Figures

List of Acronyms

DOF	Degree Of Freedom
UAV	Unmanned Aerial Vehicle
LQR	Linear Quadratic Regulator
PD	Proportional Derivative
PID	Proportional Integral Derivative
SMC	Sliding Mode Control
MPC	Model Predictive Control

List of Symbols

J_{xx}	Area moment of inertia about x-axis
J_{yy}	Area moment of inertia about y-axis
J_{zz}	Area moment of inertia about z-axis
J_r	Rotor inertia
U	Control input vector
ϕ	Roll angle about x-axis
θ	Pitch angle about y-axis
ψ	Yaw angle about z-axis

Chapter One

Introduction

This thesis work involves a simulation study of the dynamic modelling and reference-tracking control of Hexacopter UAV, which is a multirotor with six rotors. These rotors are symmetrical and fixed, and they controlled by varying the angular speed of those rotors to change the thrust and torque.

This chapter introduces the background, statement of the problem, objective and methodology of the thesis .

1.1. Background

An Aircraft is a vehicle, which is capable of flying. Depending on how they can be controlled, aircrafts can be classified as manned aerial vehicles and unmanned aerial vehicles. Manned aerial vehicles are vehicles with onboard pilots whereas unmanned aerial vehicles are one type, which can fly without a human operator but need aerodynamic force to fly on air. They may either be controlled remotely by humans or can be controlled autonomously by using onboard computer that is programmed to do a task [1].

1.1.1. Classification of unmanned aerial vehicle (UAV)

UAVs can be classified based on different factors such as depending on range of action and function of their configuration [1, 2].

1. **Based on range of action**

UAVs can classified into seven different part depending on their range of altitude and endurance as follows [2]:

(A). **High-Altitude Long-Endurance (HALE):** These types of UAVs are capable of flying altitude over 15000 m and have endurance of more than 24hrs. They are important in performing long-range reconnaissance and surveillance mission.

(B). **Medium-Altitude Long-Endurance (MALE):** These types of UAVs are capable of flying altitude between 5000m-15000m and have endurance of maximum 24hrs. These are important for surveillance mission the same as HALE.

(C). **Medium Range or Tactical UAV (TUAV):** these are capable of flying altitude between 100 and 300 km.

(D). **Close-Range UAV**: They operate in range of 100km. they are important in target designation, airfield security, crop spraying, power line inspection and traffic monitoring etc.

(E). **Mini UAV (MUAV):** They operate in the range of about 30km and have weight of 20kg.

(F). **Micro UAV (MAV):** They have wingspan of maximum 150mm. They are required to fly slowly and hover.

(G). **Nano Air Vehicle (NAV):** They have a size of 10mm. They are used for radar confusion and for ultra-short range surveillance.

2. **Based on their configuration**

Depending on the aerodynamic configuration, UAVs can be classified as follows [2, 3]:

(A). **Fixed wing UAVs:** These are UAVs which need a run-away for take off and landing. They have advantage of high cruising speed and long flying time. They can be used for long distance, long range and high altitude missions and perform scientific applications like metrological reconnaissance and environmental monitoring.

(B). **Rotary wing UAVs**: This type of UAVs can vertically take off and land in a small place without requiring a run away. They have a great advantage than the fixed wing in terms of maneuverability capabilities and hovering at a specified place. They in turn can classified as follows [1]:

(i). **Single rotors**: These UAVs have one main rotor on top and another rotor at the tail for stability which is the same as the helicopter configuration.

(ii). **Coaxial rotors:** This type have two rotors mounted to the same shaft which are rotating in opposite direction.

(iii). **Quadrotor:** This type have four rotors that are found in the corners of a cross.

(iv). **Multirotors:** These are UAVs having six or eight rotors. They are advantageous in that they can fly without a crash if one rotor fails and have a high lifting capacity that can make it to move quickly and easily [4].

1.1.2. Applications of UAVs

UAVs have the ability to replace humans and become advantageous. UAVs have many applications in the military sectors such as to observe a region and locate an enemy, to ascertain a strategic feature, to attack a mission and to do a border surveillance. However, they are also important in nonmilitary civil applications such as for forecasting a metrology, for security surveillance and inspection of power lines, for monitoring a traffic and for performing a mission such as taking photo and videos, in places where there is high human risk. Moreover, they are advantageous in commercial aerial surveillance, mineral exploration and production, and transportation of goods [3, 4].

Furthermore, UAVs have an application in earth science for cloud and Aerosol measurement, for studying global warming in measuring ice thickness and for measuring gravitational acceleration. They are also used in law enforcement and in industrial applications for pipeline inspection [1]. The rotorcraft UAV we study in this work is Hexacopter which is a multirotor rotorcraft lifted with six rotors.

1.2. Statement of the problem

Hexacopter is a highly nonlinear, unstable and multi input multi output system. Moreover, it is under actuated system with four inputs and six outputs, which are coupled each other, makes the controlling more challenging. Mostly the previous works on Hexacopter were done by using linear controllers like PID, LQR-PID, LQR-PD controllers to make the system stable. However, using these linear controllers need a linearized model and they will be linearized at hovering position. Then the Hexacopter cannot follow the path when it is out of hover [1]. In addition to these, the Hexacopter system has highly affected by disturbances like wind. For this reason, formulation of a robust method is a nonlinear controller sliding mode control (SMC), will give a good performance for tracking the path, good in disturbance rejection and increase the stability. Therefore, in this work sliding mode control (SMC) will be used for trajectory tracking control of a Hexacopter.

1.3. Objective

1.3.1. General objective

The general objective of this work is to design a mathematical model and trajectory tracking control of a Hexacopter using sliding mode control (SMC).

1.3.2. Specific objectives

- Mathematically model the dynamics of the Hexacopter
- Design the SMC controller
- Generating trajectory and Simulating the trajectory tracking using MATLAB Simulink
- Validate the results

1.4. Methodology

The methodology followed in this thesis work is described in tabular form in Table 1.

Table 1: Methodology

Methodology	**Task**
Literature review	Summarizing existing works published on the topic that are published in books, journals and articles
Modeling the system	Designing a mathematical relation of dynamics of a Hexacopter
Designing a controller	Formulating proper controller that control the system model
Simulation	Simulating the open loop system dynamics and closed loop controlling system using MATLAB Simulink
Analysis and interpretation	Analyzing and interpreting the result of simulation

1.5. Limitations

The scope of this thesis is limited to modeling and controlling the system and designing the system using a MATLAB simulation only. The hardware work is not done on this thesis because of lack of financial and material.

1.6. Thesis outline

This thesis is structured in five chapters as follows:

- Chapter Two deals with the over view of Hexacopter and review of works done before about the title.
- Chapter Three discusses about terminologies used for modeling, presents the model that is formulated using Newton Euler formalism and shows the verification of the model. It also includes the development of the controller that makes the system to track the trajectory effectively.
- Chapter Four focuses on the simulation of the system using MATLAB Simulink simulations and analyze the result.
- Chapter Five finalizes by concluding the work and suggesting a recommendation.

Chapter Two

Theoretical background and literature review

This chapter deals about the theoretical background about haxacopter and the related works done about the topic.

2.1. Hexacopter

In this paper work, the multirotor that is studied is a Hexacopter. It is a rotorcraft type UAV, which has six rotors in the vertices of a regular hexagon. These rotors are connected symmetrically to the central hub. Three of the propellers rotate in the same direction (clockwise) and the other three propellers rotate in opposite direction (counter clockwise) to balance the total system torque. When all the propellers rotate in the same direction the body will start rotating in opposite direction because of the torque created by rotors. So to remove this problem there must be anti-torque and the rotors must rotate in opposite direction.

Hexacopter is an under actuated system because it has six degree of freedom (DOF) but has four control inputs, the trust force and the aerodynamic torques. The six DOF is because six variables $x, y, z, \phi, \theta, \psi$, which are the translational and rotational components, are used to express its position in space. The translational variables x, y and z express the distance of the Hexacopter center of mass with respect to x, y and z axes whereas the rotational variables ϕ, θ, ψ are the Euler angles that tells the orientation of the Hexacopter. ϕ is the roll angle through the x axis, θ is the pitch angle through the y axis and ψ is the yaw angle along the z axis [5].

Hexacopter is controlled by using its propellers. Each of the six propellers produces an upward thrust force by pressing the air down and with the same time, they will generate a vertical movement. Whereas, in order to generate a down ward movement the speed of all the propellers must be decrease.

2.1.1. Basic movements

There are three basic movements in a Hexacopter such as roll, pitch and yaw.

2.1.1.1. Roll

Roll movement is the movement of the Hexacopter about the x-axis. As shown in Figure 2.1, it is performed by increasing/decreasing the speed of rotors 4, 5 and 6 and decreasing/increasing the speed of rotors 1, 2 and 3. Then the total thrust will remained constant and the roll rotation can be achieved by rotating around the x-axis.

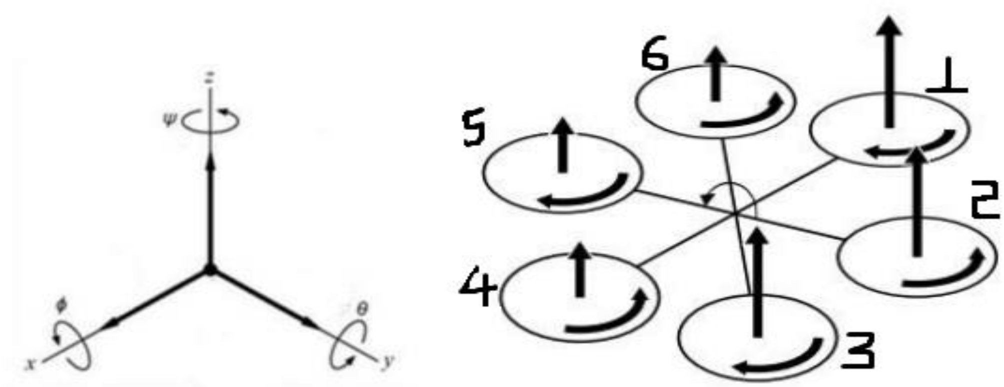


Figure 1.1: Roll Movement of Hexacopter [9]

2.1.1.2. Pitch

Pitch movement is the movement of the Hexacopter about the y-axis. It is performed through the increasing/decreasing the speed of rotors 1 and 6 and decreasing /increasing of speed of rotors 3 and 4 as shown in Figure 3. Then the total thrust will remain constant and the pitch rotation can be achieved by rotating around the y-axis.

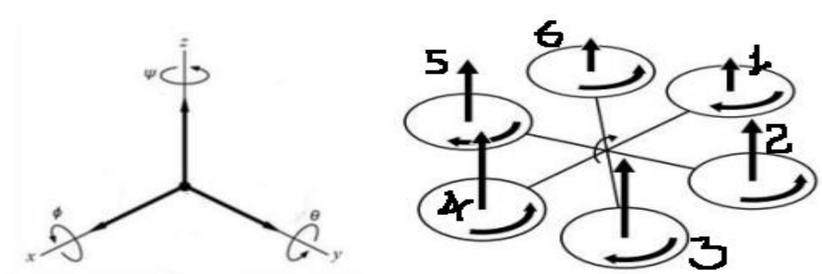


Figure 2.2: Pitch Movement of Hexacopter [9]

2.1.1.3. Yaw

Yaw movement is the movement of the Hexacopter about the z-axis or flying straight up ward. As described in Figure 4, it is performed by increasing/decreasing the angular speed of rotors rotating clockwise (1, 3, 5) and decreasing/increasing the angular speed of rotors rotating counter clockwise (2, 4, 6). Then the total thrust will remain unchanged and this makes the Hexacopter stable.

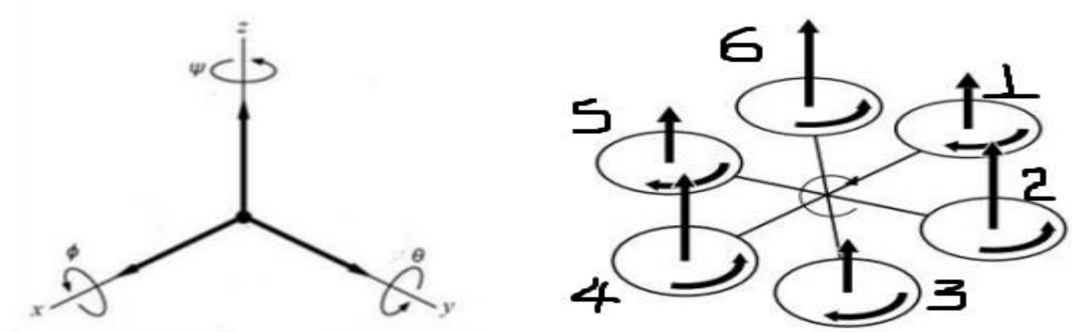


Figure 2.3: Yaw Movement of Hexacopter [9]

2.2. Literature Review

Recently multirotors have become an interesting field of research in which different papers have analyzed different aspects of Hexarotor including modeling of a rigid body, controlling and simulation of it.

Moussed, Sayouti, Medromi [5]; have analyzed on modeling of the Hexarotor using Newton Euler formalism for rotational and translational equation of motion. Proportional integral derivative (PID), Backstepping and Sliding mode controllers (SMC) have been applied for controlling the attitude, altitude and heading of a Hexarotor UAV system in space. The performance of these controllers were compared and they show that SMC and Backstepping controllers are good in stabilizing the system with great dynamic performance and are robust whereas PID is not adequate to stabilize the system and eliminate the disturbance. They recommended developing a hybrid controller such as integral action with a Backstepping controller and applying it on a real Hexarotor hardware.

A.Alaimo. etal, [6]; have used Newton Euler formalism in terms of quaternion for modeling Hexacopter in order to improve the numerical efficiency and stability of the controller algorithm.

Six PID were used as a controller for system trajectory tracking but to apply the controller quaternion error is used instead of classical linearization using Lyapunov approach.

A.Alaimo. et el. [7]; design the modeling of the Hexacopter dynamics using Newton Euler formalism. They control the Hexacopter dynamics using optimal control algorithms that are linear quadratic regulator (LQR) using PD and PID controllers to make the system more stable by linearizing the system around hovering configuration. The result has been tested by comparing the impulse disturbance response of the nonlinear dynamical system with the response of the linearized model. The result show that the system has stabilized at hovering position.

Geovanny D.C.G. et el. [8]; design the model of the system using Newton Euler formalism for the translational and rotational component dynamics. In their work sliding mode controller has been used as a controller for making the system stable. The chosen SMC algorism was implemented by applying Proportional derivative (PD) controller as sliding surface. A control strategy such as PID and SMC has been tested and compared to control the system can take off and land with the involvement of a disturbance. SMC shows a better performance in stability and have a satisfactory output in spite of nonlinearity in different operating conditions. In comparison with PID controller, SMC is more robust, having less steady state error and have less oscillation.

Tobias Magnusson [9]; wrote a thesis on Hexarotor by modelling the system using Newton Euler formalism based quaternion presentation to eliminate a singularity on system. Model predictive control is used as a controller to make the system stable and compare the result with a linear quadratic controller. Their results shows that MPC has a great tracking ability through the reference trajectory than linear quadratic.

In sum, most of the studies in the literature reviewed above have focused on stability and controlling the altitude. However, in this work a nonlinear controller SMC is used for controlling the whole system altitude, attitude, heading and position of the Hexarotor in tracking a specific reference. For a highly nonlinear, coupled and unstable Hexarotor system, SMC has better result for making the system more stable, to get a better tracking response with a minimum error and fast convergence.

Chapter Three

System modeling and Controller design

In this chapter, the kinematic and dynamic model of the Hexacopter and the control design for the Hexacopter system will be discussed.

3.1. Kinematic modeling

Kinematics is a branch of mechanics that focus on the motion of objects without the reference to the cause of the motion. To express it there are two frames of references in a rigid motion: body fixed frame and earth inertial frame of reference. In inertial frame, the absolute linear position of the body is expressed as $X_{E,}\ Y_E and\ Z_E$. Whereas the body fixed frame ($X_B, Y_B and\ Z_B$) which center at center of gravity of Hexacopter. The orientation of the body fixed frame can be expressed by Euler angle as roll (ϕ), pitch (θ) and yaw (ψ) expressing rotation about x, y and z axes respectively.

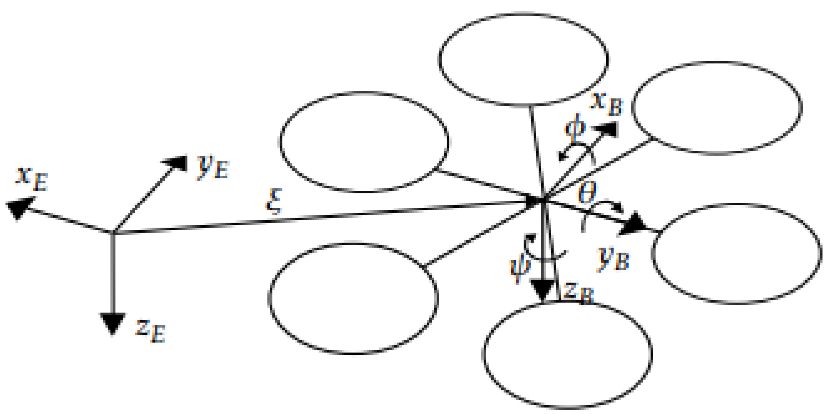


Figure 3.1: The two frames of references [9]

The body-fixed frame's position in the earth-fixed frame can be described by using the vector $\xi = [x\ y\ z]T$ and its orientation, attitude and heading by the vector $\eta = [\phi\ \theta\ \psi]T$ where the angles $\phi, \theta\ and\ \psi$ are called the roll, pitch and yaw respectively. To bring the body-fixed frame into coincidences with the earth-fixed frame the following rotations are considered.

To describe the rotation from body to earth frame first rotate the z- axis by yaw angle ψ and subsequently rotate the new frame about y- axis by pitch angle θ then rotate the new frame about x-axis by roll angle ϕ.

Rotation about the earth fixed frame z-axis about a yaw angle ψ and forming the new frame A

$$R(\psi) = \begin{bmatrix} cos\psi & sin\psi & 0 \\ -sin\psi & cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3.1}$$

Then resulting new frame A is rotated about y-axis by pitch angle θ to form a new frame A' which is expressed by

$$R(\theta) = \begin{bmatrix} cos\theta & 0 & -sin\theta \\ 0 & 1 & 0 \\ sin\theta & 0 & cos\theta \end{bmatrix} \tag{3.2}$$

Then lastly rotation of new frame A' about x-axis by roll angle ϕ which result in the body frame. This rotation is given by

$$R(\phi) = \begin{vmatrix} 1 & 0 & 0 \\ 0 & cos\phi & sin\phi \\ 0 & -sin\phi & cos\phi \end{vmatrix} \tag{3.3}$$

Finally, the rotation matrix to transform earth frame to body frame will become the multiplication of the above rotations matrixes given by

$$R_B^E = R(\psi)R(\theta)R(\phi)$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\phi & sin\phi \\ 0 & -sin\phi & cos\phi \end{bmatrix} \begin{bmatrix} cos\theta & 0 & -sin\theta \\ 0 & 1 & 0 \\ sin\theta & 0 & cos\theta \end{bmatrix} \begin{bmatrix} cos\psi & sin\psi & 0 \\ -sin\psi & cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3.4}$$

After multiplication the rotation matrix for transformation of earth frame to body frame is expressed as

$$R_B^E = \begin{bmatrix} cos\psi cos\theta & cos\phi sin\psi & -sin\theta \\ cos\psi sin\phi sin\theta - sin\psi cos\phi & sin\phi sin\theta sin\psi + cos\phi cos\psi & sin\phi cos\theta \\ cos\phi cos\psi sin\theta + sin\phi sin\psi & cos\phi sin\theta sin\psi - cos\psi sin\phi & cos\phi cos\theta \end{bmatrix} \tag{3.5}$$

In order to transform quantities defined in the body frame to earth fixed frame, The inverse of the rotation matrix expressed by the transpose of the R_B^E is used and given by

$$R_E^B = {R_B^E}^T = \begin{bmatrix} cos\psi cos\theta & cos\psi sin\phi sin\theta - sin\psi cos\phi & cos\phi cos\psi sin\theta + sin\phi sin\psi \\ cos\phi sin\theta & sin\phi sin\theta sin\psi + cos\phi cos\psi & cos\phi sin\theta sin\psi - cos\psi sin\phi \\ -sin\theta & sin\phi cos\theta & cos\phi cos\theta \end{bmatrix} \quad (3.6)$$

This rotation matrix has an importance for transforming forces that are defined in a given frame to a different or similar frame of reference. For instance, some forces such as gravitational force are defined in earth frame and some forces like forces produced by propeller are defined in a body frame.

The linear velocity in the body frame is expressed as vector V_B= [u v w] T and its transformation to the earth frame is given by

$$\dot{\xi}_E = R_E^B V_B \quad (3.7)$$

where $\dot{\xi}_E = [\dot{x}\ \dot{y}\ \dot{z}]T$ is time derivative of position in earth frame

In order to relate the body angular velocity, which can be expressed as $\omega_B = [p\ q\ r]T$ where $p, q\ and\ r$ are the rotations around $x_B, y_B\ and\ z_B$ axises respectively, with angular velocity in the earth fixed frame $\dot{\eta}_E$ = [ϕ θ ψ]T, yaw transformation by angle ψ is first applied and then will be followed by pitch transformation by angle θ and finally roll transformation by angle ϕ will be applied.

Initially $\omega_E = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ that tells there is no rotation in the earth frame

First rotation about z axis by yaw angle $R(\psi)$ forming a new reference frame A' and is given by

$$\omega_{A'} = R(\psi)(\omega_E) + \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix} \quad (3.8)$$

Next rotating the new frame A' about y-axis by pitch angle $R(\theta)$ to a frame A''

$$\omega_{A''} = R(\theta) \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix} + \begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix} = \begin{bmatrix} -\dot{\psi} sin\theta \\ \dot{\theta} \\ \dot{\psi} cos\theta \end{bmatrix} \quad (3.9)$$

Then rotating frame A'' about x axis by roll angle $R(\phi)$ to the body frame

$$\omega_B = R(\boldsymbol{\phi})\begin{bmatrix}-\dot{\psi}sin\theta\\ \dot{\theta}\\ \dot{\psi}cos\theta\end{bmatrix} + \begin{bmatrix}\dot{\phi}\\0\\0\end{bmatrix} = \begin{bmatrix}\dot{\phi}-\dot{\psi}sin\theta\\ \dot{\theta}cos\phi+\dot{\psi}sin\phi cos\theta\\ -\dot{\theta}sin\phi+\dot{\psi}cos\phi cos\theta\end{bmatrix} = R_{vel_B}^{\ E}\dot{\eta}_E \tag{3.10}$$

where $R_{vel_B}^{\ E}$ is the rotation matrix for transforming the angular velocity Euler angles from earth fixed frame to body frame. It is defined from the above equation (3.10) that $\dot{\eta}_E = [\dot{\phi}, \dot{\theta}, \dot{\psi}]T$ and substituting it to $\omega_B = R_{vel_B}^{\ E}\dot{\eta}_E$.

$$\begin{bmatrix}\dot{\phi}-\dot{\psi}sin\theta\\ \dot{\theta}cos\phi+\dot{\psi}sin\phi cos\theta\\ -\dot{\theta}sin\phi+\dot{\psi}cos\phi cos\theta\end{bmatrix} = R_{vel_B}^{\ E}\begin{bmatrix}\dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix} \tag{3.11}$$

Then we get

$$R_{vel_B}^{\ E} = \begin{bmatrix}1 & 0 & -sin\theta\\ 0 & cos\phi & sin\phi cos\theta\\ 0 & -sin\phi & cos\phi cos\theta\end{bmatrix} \tag{3.12}$$

On the other hand, the inverse transformation for transforming angular velocity from body frame to earth fixed frame becomes

$$R_{Vel_E}^{\ B} = \begin{bmatrix}1 & sin\phi tan\theta & cos\phi tan\theta\\ 0 & cos\phi & -sin\phi\\ 0 & \frac{sin\phi}{cos\theta} & \frac{cos\phi}{cos\theta}\end{bmatrix} \tag{3.13}$$

This relates the time derivative of Euler angles $\dot{\eta}_E$ with angular velocity ω_B from equation (3.10) which give by

$$\dot{\eta}_E = R_{Vel_E}^{\ B}\,\omega_B \tag{3.14}$$

Here the orientation shows that $R_{Ve_E}^{\ B}$ is defined if and only if $\theta \neq \frac{\Pi}{2} + K\Pi$ where K ϵ Z.

3.2. Dynamic modeling of Hexacopter

The dynamic model of the Hexacopter has translational components (altitude, x position and y position) and rotational components (roll, pitch and yaw). In order to derive the equation of motion we begin with stating certain assumptions [5]:

- Hexacopter is a type of rigid body;
- It has symmetrical structure;

- The trusts and force are proportional to the square of speed of the rotors;
- The six rotors are found in vertices of a regular hexagon.

In order to drive the dynamic model for position and orientation of the Hexacopter, Newton Euler general formalism is applied [5].

$$\begin{bmatrix} mJ_{3x3} & 0_{3x3} \\ 0_{3x3} & J \end{bmatrix} \begin{bmatrix} \dot{V} \\ \dot{\omega} \end{bmatrix} + \begin{bmatrix} \omega \ X \ mV \\ \omega \ X \ J\omega \end{bmatrix} = \begin{bmatrix} \sum F \\ \sum M \end{bmatrix} \tag{3.15}$$

Where m is the mass of the body in Kg , J is the inertia in Nms^2, V is the linear velocity in m/s, ω is the angular velocity in rad/sec, F is the force acting on body in N and M is the torque affecting body of the Hexacopter in *Nm*, 0_{3x3} is zero matrix with size 3, J_{3x3} is a unit matrix with size 3.

3.2.1. Translational Dynamics

In the earth frame of reference, the translational dynamics of the Hexarotor can be calculated by using newton's second law of motion as

$$F = ma = m\ddot{\xi} = m \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} \tag{3.16}$$

Where F force acting on the body

m mass of Hexacopter body

a Acceleration in the earth frame

$\ddot{\xi}$ Time double derivative of the position in reference of the earth frame.

By using rotational matrix as derived in previous section, the above equation (3.5) can be expressed in body fixed frame as

$$R_B^E F^B = mR_B^E a^B = mR_B^E \dot{V} \tag{3.17}$$

V is the velocity of the body and its time derivative is in body frame. Then the final expression of the translational dynamics in the body fixed frame becomes

$$F^B = m\dot{V} + \omega \ x \ mV \tag{3.18}$$

3.2.2. Rotational dynamics

The rotational dynamics of a Hexacopter is derived by using Euler's second axiom in the inertial frame that states the time derivative of the angular momentum is equal with the external torques applied on the body.

$$M_E = \dot{L}_E \tag{3.19}$$

where $\dot{L}_E$ is the angular momentum of the body in the earth frame and

M_E denotes the sum total of external applied moments in the body frame

Using rotational matrix it can be transformed to body reference frame as

$$R_B^E M_E = R_B^E \dot{L}_E = R_B^E \left(\frac{dJ\omega}{dt}\right) \tag{3.20}$$

where the inertial matrix J , defined by $J = \begin{bmatrix} J_{xx} & 0 & 0 \\ 0 & J_{yy} & 0 \\ 0 & 0 & J_{zz} \end{bmatrix}$ is a diagonal matrix in which the non-diagonal elements are zero because of the symmetry of the Hexacopter.

Finally, the rotational dynamics in the body frame gives

$$M_B = J\dot{\omega} + \omega\ x\ J\omega \tag{3.21}$$

3.2.3. Applied forces on the Hexacopter

1. Gravity

Naturally most forces, torques and other factors exist in the body frame whereas gravity is the only force that naturally exist in the earth inertial frame. Depending on Euler's first axiom, gravitational force is a force that happens at the Hexacopter center of gravity. It is located in the downward direction and expressed as

$$F_g = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} \tag{3.22}$$

In body frame is expressed as

$$F_g^B = R_B^E \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} = -mg \begin{bmatrix} -sin\theta \\ sin\phi cos\theta \\ cos\phi cos\theta \end{bmatrix} \tag{3.23}$$

where m is the mass of the body and g is the acceleration due to gravity

2. **Thrust force**

Thrust force is the force that happens due to the propellers that makes the body to lift upward or makes to move in the direction of motion. It is located in the direction positive z- axis.

The lift force is defined as the summation of thrusts of the six propellers

$$F_T = T_1 + T_2 + T_3 + T_4 + T_5 + T_6 \tag{3.24}$$

Where T_i is the thrust from propellers

Since the origin of thrust is a propeller, the generated thrust T_i from speed of propeller Ω_i is expressed as

$$T_i = b\Omega_i^2 = C_T \rho A r^2 \Omega_i^2 \tag{3.25}$$

where Ω_i is angular speed of propeller i,

b is propeller specific constant expressed by $C_T \rho A r^2$,

C_T is thrust coefficient,

ρ is air density,

A is rotor disk area and

r is rotor radius.

Finally, it is obtained that the total thrust force expressed in the body frame becomes

$$\begin{aligned} F_T &= \sum_{i=1}^{6} T_i \\ &= b\Omega_1^2 + b\Omega_2^2 + b\Omega_3^2 + b\Omega_4^2 + b\Omega_5^2 + b\Omega_6^2 \end{aligned} \tag{3.26}$$

3.2.4. Applied torques

1. **Torque induced by thrust forces**

As the propellers are not found at the center of gravity, they will create a torque in different axis of rotation. As seen in the Figure 3.2, each rotor creates a moment with direction opposite to rotation of the rotor.

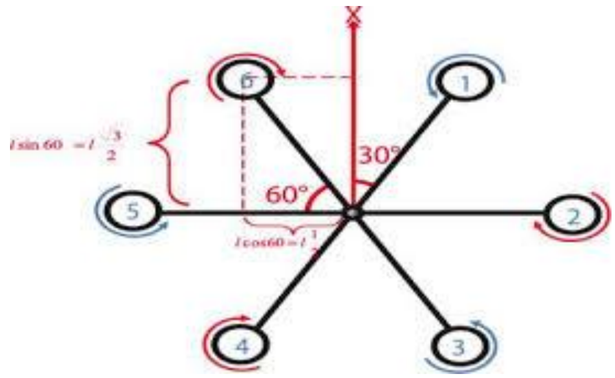


Figure 3.2: Hexacopter rotor distance from center of gravity [5]

Around the x-axis, by applying a right hand rule in relation with the axes of rotation, the applied moment produced by propellers M_r becomes

$$\text{Roll torque} \quad M_r = -\frac{l}{2}T_1 - lT_2 - \frac{l}{2}T_3 + \frac{l}{2}T_4 + lT_5 + \frac{l}{2}T_6 \tag{3.27}$$

where T_i is the thrust force, which is equal to $b\Omega_i^2$, and

l is the length of the arm.

Around the y-axis by applying right hand rule, rotor 2 and 5 do not have any moment on y axis, thus the applied torque moment M_P gives

$$\text{Pitch torque} \quad M_p = \frac{\sqrt{3}}{2}lT_1 - \frac{\sqrt{3}}{2}lT_3 - \frac{\sqrt{3}}{2}lT_4 + \frac{\sqrt{3}}{2}lT_6 \tag{3.28}$$

The torque around the z-axis is the result of action and reaction forces. That is when the propellers rotate, they exert a torque through the air frame. From [9] the reaction torque Q_i of the propeller i gives

$$Q_i = d\Omega_i^2 \tag{3.29}$$

where d is a specific constant of the propeller.

$$\text{Yaw torque} \quad M_y = -T_1 + T_2 - T_3 + T_4 - T_5 + T_6 \tag{3.30}$$

Finally, the total applied torque through the induced thrust forces of the propeller in relation to the speed of the propellers is written as

$$M_r = -\frac{1}{2}bl\Omega_1^2 - bl\Omega_2^2 - \frac{1}{2}bl\Omega_3^2 + \frac{1}{2}bl\Omega_4^2 + bl\Omega_5 + \frac{1}{2}bl\Omega_6^2$$

$$M_p = \frac{\sqrt{3}}{2}bl\Omega_1^2 - \frac{\sqrt{3}}{2}bl\Omega_3^2 - \frac{\sqrt{3}}{2}bl\Omega_4^2 + \frac{\sqrt{3}}{2}bl\Omega_6^2 \qquad (3.31)$$

$$M_y = -d\Omega_1^2 + d\Omega_2^2 - d\Omega_3^2 + d\Omega_4^2 - d\Omega_5^2 + d\Omega_6^2$$

2. Gyroscopic effect from propellers

Gyroscopic effect produced by the propeller rotation through the shaft called spin and rotation of the airframe called precision. Gyroscopic torque from propeller is given by

$$M_G = \omega \times J_p \begin{bmatrix} 0 \\ 0 \\ -(1)^i\Omega_i \end{bmatrix} \qquad (3.32)$$

where ω is the speed of propeller and J_p is propellers inverse matrix

$$M_G = \omega \times \begin{bmatrix} 0 \\ 0 \\ J_{P,zz}(-1)^i\Omega_i \end{bmatrix} = \begin{bmatrix} qJ_{p,zz}(-1)^i\Omega_i \\ -pJ_{p,zz}(-1)^i\Omega_i \\ 0 \end{bmatrix} \qquad (3.33)$$

Which is the same as $$M_G = \begin{bmatrix} qJ_{p,zz}\Omega_r \\ -pJ_{p,zz}\Omega_r \\ 0 \end{bmatrix} \qquad (3.34)$$

where Ω_r is the resultant angular velocity expressed as $\Omega_r = \sum_1^6(-1)^i\Omega_i$

3.3. Final system modeling

3.3.1. Control input vector *U*

By controlling the rotational inputs, one can control the rotors movement of the vehicle. The input for the Hexacopter system is chosen as the rotational speed of all the six propellers. These rotational speeds have a relation with the aerodynamic thrust force and torques. For this system there are four control inputs defined by

$$U = [U_1 \;\; U_2 \;\; U_3 \;\; U_4]$$

where $U_1 = b\left({\Omega_1}^2 + {\Omega_2}^2 + {\Omega_3}^2 + {\Omega_4}^2 + {\Omega_5}^2 + {\Omega_6}^2\right) = F_T$ (3.35)

$$U_2 = -\frac{1}{2}bl\Omega_1^2 - bl\Omega_2^2 - \frac{1}{2}bl\Omega_3^2 + \frac{1}{2}bl\Omega_4^2 + bl\Omega_5 + \frac{1}{2}bl\Omega_6^2 = M_r \tag{3.36}$$

$$U_3 = \frac{\sqrt{3}}{2}bl\Omega_1^2 - \frac{\sqrt{3}}{2}bl\Omega_3^2 - \frac{\sqrt{3}}{2}bl\Omega_4^2 + \frac{\sqrt{3}}{2}bl\Omega_6^2 = M_p \tag{3.37}$$

$$U_4 = -d\Omega_1^2 + d\Omega_2^2 - d\Omega_3^2 + d\Omega_4^2 - d\Omega_5^2 + d\Omega_6^2 = M_y \tag{3.38}$$

From the above equation F_T corresponds to is the trust force defined at equation (3.26) and M_r, M_p and M_y are the roll, pitch and yaw torques respectively defined at equation (3.31).

The control inputs in equation (3.35) to equation (3.38) can be expressed in matrix form as

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} b & b & b & b & b & b \\ -bl/2 & -bl & -bl/2 & bl/2 & bl & bl/2 \\ \sqrt{3}bl/2 & 0 & -\sqrt{3}bl/2 & -\sqrt{3}bl/2 & 0 & \sqrt{3}bl/2 \\ -d & d & -d & d & -d & d \end{bmatrix} \begin{bmatrix} \Omega_1{}^2 \\ \Omega_2{}^2 \\ \Omega_3{}^2 \\ \Omega_4{}^2 \\ \Omega_5{}^2 \\ \Omega_6{}^2 \end{bmatrix} \tag{3.39}$$

From the above matrix equation (3.38), U_1 is the control input for the altitude that it is the sum of the six rotors speed, which causes an upward thrust force. U_2 is the difference in rotors thrust 1,2,3 and 4,5,6 which makes a roll movement. U_3 is a control input which represents a difference of rotors thrust 1,6 and 3,4 which is responsible for a pitch rotation. Finally, U_4 is the difference in rotors torque between the three clockwise rotating rotors and the three counter clockwise rotating rotors that generates a yaw rotation.

If the rotor velocity is needed to be calculated from the control inputs, the inverse relation between the rotor's speed and control inputs is needed, which can be done by inverting the above matix in equation (3.39).

$$\begin{bmatrix} \Omega_1{}^2 \\ \Omega_2{}^2 \\ \Omega_3{}^2 \\ \Omega_4{}^2 \\ \Omega_5{}^2 \\ \Omega_6{}^2 \end{bmatrix} = \begin{bmatrix} \frac{1}{6b} & \frac{1}{3bl} & 0 & \frac{-1}{6d} \\ \frac{1}{6b} & \frac{1}{6bl} & \frac{-\sqrt{3}}{6bl} & \frac{1}{6d} \\ \frac{1}{6b} & \frac{-1}{6bl} & \frac{-\sqrt{3}}{6bl} & \frac{-1}{6d} \\ \frac{1}{6b} & \frac{-1}{3bl} & 0 & \frac{1}{6d} \\ \frac{1}{6b} & \frac{-1}{6bl} & \frac{\sqrt{3}}{6bl} & \frac{-1}{6d} \\ \frac{1}{6b} & \frac{1}{6bl} & \frac{\sqrt{3}}{6bl} & \frac{1}{6d} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} \tag{3.40}$$

3.3.2. Translational equation of motion

The complete translational dynamics become

$$\sum F = ma = m\ddot{\xi} = m[\ddot{x} \quad \ddot{y} \quad \ddot{z}]^T \tag{3.41}$$

As seen above we have applied gravitational force and thrust forces and combining them, we have got

$$m\begin{bmatrix}\ddot{x}\\ \ddot{y}\\ \ddot{z}\end{bmatrix} = F_g + F_T \tag{3.42}$$

Substituting the values for F_g and F_ω from equation (3.22) and (3.26), it becomes

$$m\begin{bmatrix}\ddot{x}\\ \ddot{y}\\ \ddot{z}\end{bmatrix} = \begin{bmatrix}0\\ 0\\ -mg\end{bmatrix} + R_E^B\begin{bmatrix}0\\ 0\\ b\sum\Omega_i^2\end{bmatrix} \tag{3.43}$$

Finally rewriting the above equation, the acceleration of the translational components gives

$$\ddot{x} = \frac{1}{m}(cos\phi cos\psi sin\theta + sin\phi sin\psi)U_1 \tag{3.44}$$

$$\ddot{y} = \frac{1}{m}(cos\phi sin\theta sin\psi - sin\phi cos\psi)U_1 \tag{3.45}$$

$$\ddot{z} = \frac{1}{m}(cos\theta cos\phi)U_1 - g \tag{3.46}$$

3.3.3. Rotational equation of motion

Combining the above torque equations, we get

$$J\dot{\omega} + \omega \times J\omega = \sum M \tag{3.47}$$

$$J\dot{\omega} = M_A + M_G - \omega \times J\omega \tag{3.48}$$

where M_A is the torque through propellers equal with $[lU_2 \; lU_3 \; U_4]T$

Substituting the value for M_G in equation (3.34), it becomes

$$J\begin{bmatrix}\ddot{\phi}\\ \ddot{\theta}\\ \ddot{\psi}\end{bmatrix} = \begin{bmatrix}lU_2\\ lU_3\\ U_4\end{bmatrix} + \begin{bmatrix}qJ_r(\Omega_r)\\ -pJ_r(\Omega_r)\\ 0\end{bmatrix} - \begin{bmatrix}\dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix} \times J\begin{bmatrix}\dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix} \tag{3.49}$$

Finally, the total rotational dynamics of a Hexacopter becomes

$$\ddot{\phi} = \frac{1}{J_{xx}}(\dot{\theta}\dot{\psi}(J_{yy} - J_{zz}) + J_r\Omega_r\dot{\theta} + lU_2) \tag{3.50}$$

$$\ddot{\theta} = \frac{1}{J_{yy}}(\dot{\phi}\dot{\psi}(J_{zz} - J_{xx}) - J_r\Omega_r\dot{\phi} + lU_3) \tag{3.51}$$

$$\ddot{\psi} = \frac{1}{J_{zz}}(\dot{\phi}\dot{\theta}(J_{xx} - J_{yy}) + U_4) \tag{3.52}$$

Finally, the complete nonlinear dynamic model of a Hexacopter becomes

$$\begin{aligned}
\ddot{x} &= \frac{1}{m}[cos\phi cos\psi sin\theta + sin\phi sin\psi]U_1 \\
\ddot{y} &= \frac{1}{m}[cos\phi sin\theta sin\psi - sin\phi cos\psi]U_1 \\
\ddot{z} &= \frac{1}{m}[cos\theta cos\phi]U_1 - g \\
\ddot{\phi} &= \dot{\theta}\dot{\psi}\left(\frac{J_{yy}-J_{zz}}{J_{xx}}\right) + \frac{J_r\Omega_r\dot{\theta}}{J_{xx}} + \frac{l}{J_{xx}}U_2 \\
\ddot{\theta} &= \dot{\psi}\dot{\phi}\left(\frac{J_{zz}-J_{xx}}{J_{yy}}\right) - \frac{J_r\Omega_r\dot{\phi}}{J_{yy}} + \frac{l}{J_{yy}}U_3 \\
\ddot{\psi} &= \dot{\theta}\dot{\phi}(\frac{J_{xx}-J_{yy}}{J_{zz}}) + \frac{1}{J_{zz}}U_4
\end{aligned} \tag{3.53}$$

3.3.4. State space representation

First defining the state vector of the Hexacopter

$$X = [x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ x_7\ x_8\ x_9\ x_{10}\ x_{11}\ x_{12}]T \tag{3.54}$$

This state vector is associated with the Hexacopter degree of freedom

$$X = [x\ \dot{x}\ y\ \dot{y}\ z\ \dot{z}\ \phi\ \dot{\phi}\ \theta\ \dot{\theta}\ \psi\ \dot{\psi}]T \tag{3.55}$$

As it is shown above, this state vectors define the position of the Hexacopter and its linear and angular velocities.

Now using the translational equations of model from equation (3.44) to (3.46) and rotational equations (3.50) to (3.52), the full mathematical model can be represented by a state space representation as follows

$$\dot{x}_1 = \dot{x} = x_2$$

$$\dot{x}_2 = \ddot{x} = \frac{1}{m}[cos\phi cos\psi sin\theta + sin\phi sin\psi]U_1$$

$$\dot{x}_3 = \dot{y} = x_4$$

$$\dot{x}_4 = \ddot{y} = \frac{1}{m}[cos\phi sin\theta sin\psi - sin\phi cos\psi]U_1$$

$$\dot{x}_5 = \dot{z} = x_6$$

$$\dot{x}_6 = \ddot{z} = \frac{1}{m}(cos\theta cos\phi)U_1 - g \tag{3.56}$$

$$\dot{x}_7 = \dot{\phi} = x_8$$

$$\dot{x}_8 = \ddot{\phi} = \dot{\theta}\dot{\psi}\left(\frac{J_{yy} - J_{zz}}{J_{xx}}\right) + \frac{J_r \Omega_r \dot{\theta}}{J_{xx}} + \frac{l}{J_{xx}}U_2$$

$$\dot{x}_9 = \dot{\theta} = x_{10}$$

$$\dot{x}_{10} = \ddot{\theta} = \dot{\psi}\dot{\phi}\left(\frac{J_{zz} - J_{xx}}{J_{yy}}\right) - \frac{J_r \Omega_r \dot{\phi}}{J_{yy}} + \frac{l}{J_{yy}}U_3$$

$$\dot{x}_{11} = \dot{\psi} = x_{12}$$

$$\dot{x}_{12} = \ddot{\psi} = \dot{\theta}\dot{\phi}(\frac{J_{xx} - J_{yy}}{J_{zz}}) + \frac{1}{J_{zz}}U_4$$

$$f(X,U) = \begin{bmatrix} x_2 \\ \frac{1}{m}\ [cos\phi cos\psi sin\theta + sin\phi sin\psi]\ U_1 \\ x_4 \\ \frac{1}{m}[cos\phi sin\theta sin\psi - sin\phi cos\psi]U_1 \\ x_6 \\ \frac{1}{m}(cos\theta cos\phi)U_1 - g \\ x_8 \\ \dot{\theta}\dot{\psi}\left(\frac{J_{yy} - J_{zz}}{J_{xx}}\right) + \frac{J_r \Omega_r \dot{\theta}}{J_{xx}} + \frac{l}{J_{xx}}U_2 \\ x_{10} \\ \dot{\psi}\dot{\phi}\left(\frac{J_{zz} - J_{xx}}{J_{yy}}\right) - \frac{J_r \Omega_r \dot{\phi}}{J_{yy}} + \frac{l}{J_{yy}}U_3 \\ x_{12} \\ \dot{\theta}\dot{\phi}(\frac{J_{xx} - J_{yy}}{J_{zz}}) + \frac{1}{J_{zz}}U_4 \end{bmatrix} \tag{3.57}$$

3.4. Dynamic system model verification

Previously it has been stated that a Hexacopter has six degree of freedom and controlled by speed of propellers. The translational and rotational components of a Hexacopter are modeled based on the Newton Euler formalism. To check if the designed model has meet the principle of the Hexacopter's operation and response of the given constant inputs, MATLAB Simulink model scheme is used.

For the system to be as arbitrary as possible with respect to the number of rotors, numerical values for parameters in Table 2 has been used in simulation.

Table 2: The values for parameters used in the simulation

Symbol	Description	Numerical values	Sources
m	Mass of the body	2.1 kg	[Tobias,2014]
l	Length of the arm	0.23 m	[Tobias,2014]
J_{xx}	Moment of inertia along x-axis	0.0038 kgm^2	[Tobias,2014]
J_{yy}	Moment of inertia along y-axis	0.0038 kgm^2	[Tobias,2014]
J_{zz}	Moment of inertia along z-axis	0.0071 kgm^2	[Tobias,2014]
g	Acceleration due to gravity	9.8 m/s^2	[Tobias,2014]
J_r	Rotor inertia	0.0008 kgm^2	[Tobias,2014]
b	Thrust constant	0.01458 Ns^2	[Tobias,2014]
d	Drag factor	0.001037Nms	[Tobias,2014]

Simulink block

The Simulink block diagram for the designed open loop dynamic model of a Hexacopter is shown in figure (3.3) below. Simulink model has different part. The first part of the Simulink block shows the speed constants for the six propellers of the rotors that are important for calculating the torques of the four control inputs of Hexacopter. The value for speed constants is chosen randomly. Whereas, the second part of block is used for converting the speed values to torque input values in which this input values drives the motion of the multirotor.

The third part is the main component of the Simulink in which the formulated dynamic mathematical model of the Hexacopter is expressed using a MATLAB function block. The outputs of this block shows the state derivative of both the translational components $(\ddot{x}, \ddot{y}, \ddot{z})$and rotational $(\ddot{\phi}, \ddot{\theta}, \ddot{\psi})$acceleration components of the system model and the integral is applied to all components to get the state vector components$(x, y, z, \phi, \theta, \psi)$.

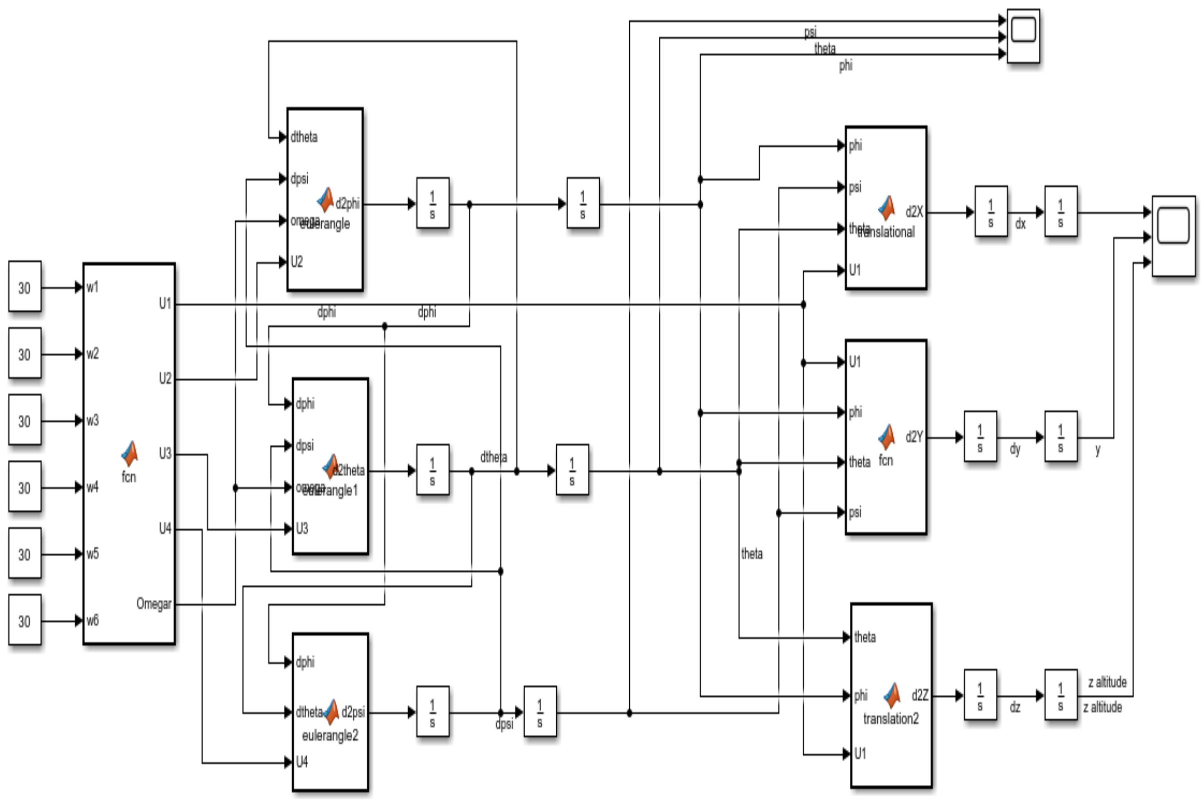
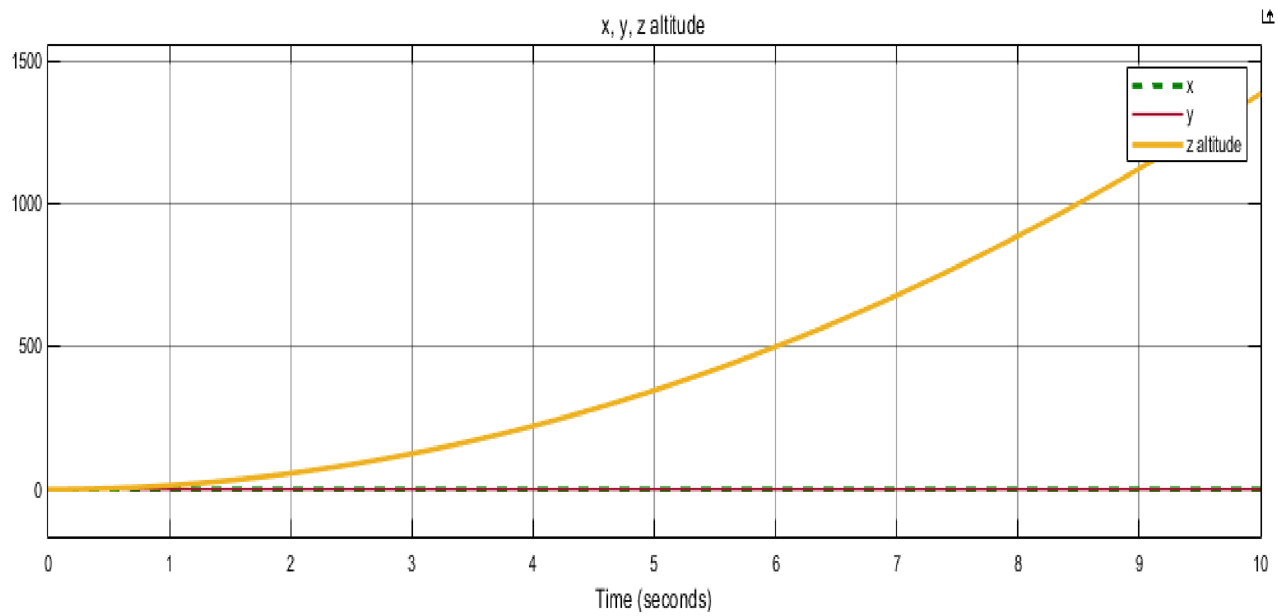
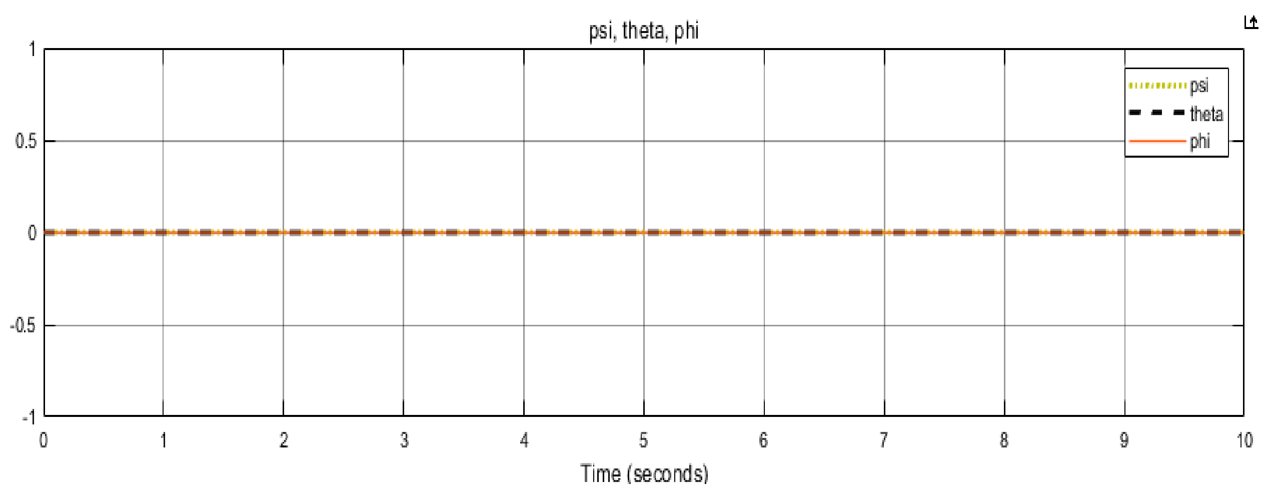


Figure 3.3: Simulink block diagram of the open loop uncontrolled model

By applying the parameters specified in the table (2) above, the output of the open loop system without a controller as seen in the scope will look like in the figure below. This result be obtained when applying the same amount of fixed rotating speed constants of the rotors having a value of 30 rpm for each of six rotor$(w_1 = w_2 = w_3 = w_4 = w_5 = w_6 = 30rpm)$.



(a)



(b)

Figure 3.4: Simulation result of open loop uncontrolled model with $w_1 = w_2 = w_3 = w_4 = w_5 = w_6 = 30rpm$ (a) translational components and (b) rotational components

From the above two Figures (figure 3.4a and 3.4b) it is clear to see that when all the six rotors are rotating with in the same propeller speed, all the parameters except the altitude becomes zero. This shows that the Hexacopter is flying in upward direction without any movement in the other direction and without any effect of gravitational force. Moreover, varying the speed of the six rotors, it also produces a roll, pitch and yaw rotation. This proves the correctness of the designed dynamic model of the Hexacopter.

The Figure below show that when all the rotors are at an equilibrium position or the six rotors are at 0 rpm then only the altitude will be down ward because of the gravitational effect of the earth.

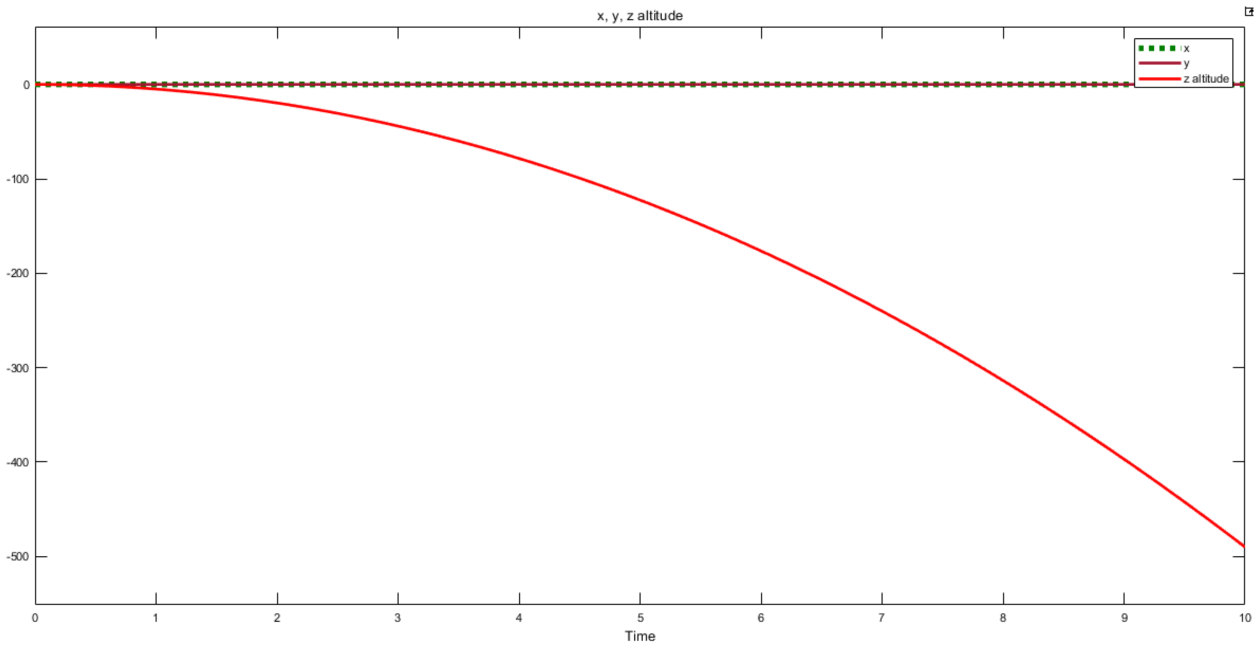


Figure 3.5: Open loop uncontrolled simulation result with($w_1 = w_2 = w_3 = w_4 = w_5 = w_6 = 0 rpm$)

3.5. Controller design

Hexarotor is a system that has a highly nonlinear and unstable system. Moreover, a proper control system is needed to make it stable. In this thesis work, a sliding mode controller (SMC) is used for making Hexarotor track a given trajectory.

3.5.1. Block diagram for controlling system

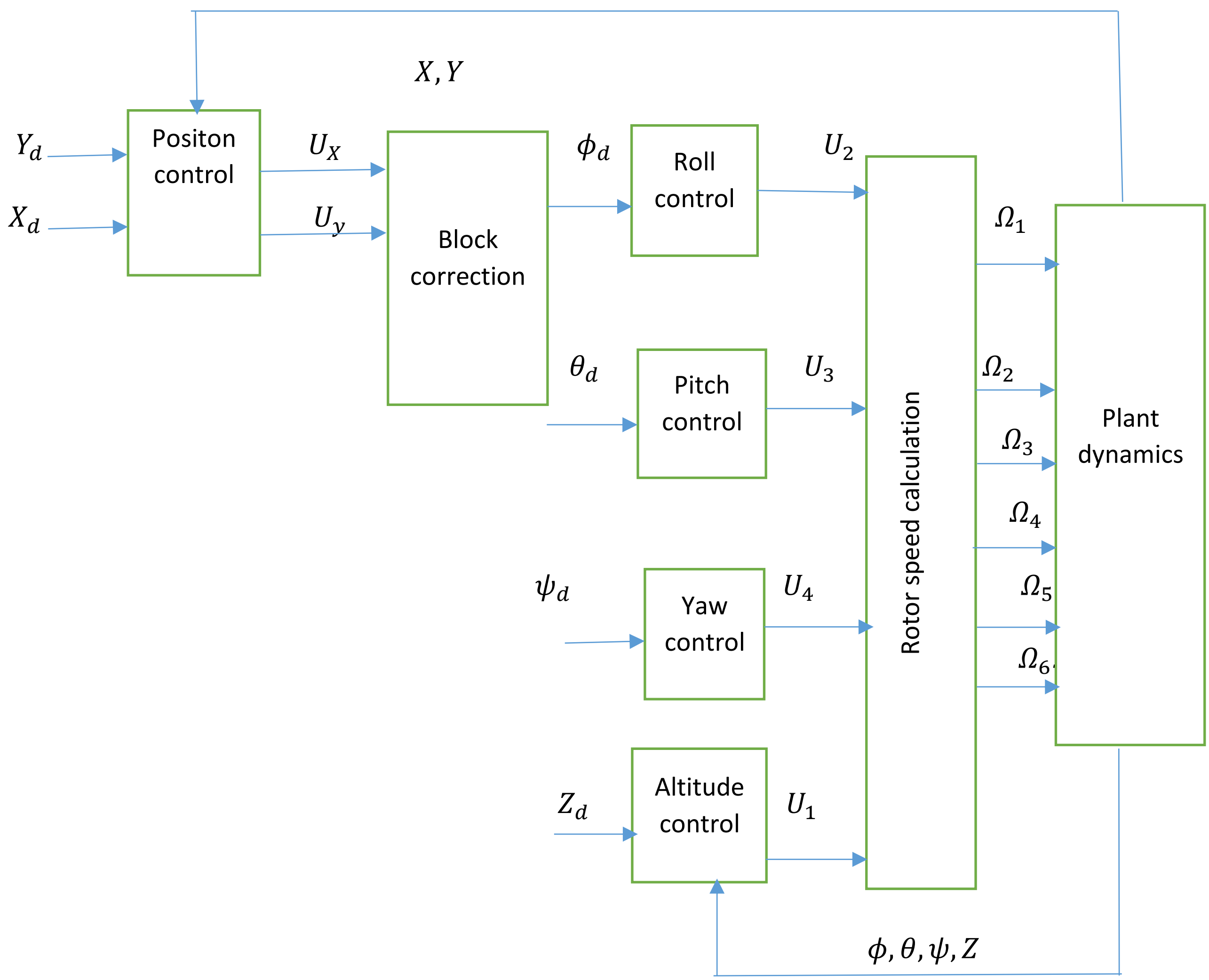


Figure 3.6: General block diagram of controlling system

From the block diagram in Figure 3.6, the control loop has the inner loop and outer loop. The inner loop has four control laws which are the roll control (ϕ), pitch control (θ), yaw control (ψ) and altitude control (Z). In addition, the outer loop has two position control laws.

Since the Hexacopter is under actuated system, it needs a six control dynamics to track the desired trajectories and to regulate roll and pitch angles at the same time [10]. To do this the outer loop should produce a desired force to control the translations of x and y. Then the outer loop controls the roll and pitch dynamics. From the block above, a block corrector is used to generate the desired value for the roll and pitch angles ϕ_d and θ_d. This is done by analytical inversion as follows

From the dynamic model of translational component

$$\text{Let} \qquad U_x = [cos\phi cos\psi sin\theta + sin\phi sin\psi]U_1 \tag{3.58}$$

$$U_y = (cos\phi sin\theta sin\psi - sin\phi cos\psi)U_1 \tag{3.59}$$

To simplify and get the expression for ϕ_d and θ_d,

first make the above equation in matrix form

$$\begin{bmatrix} sin\psi & -cos\psi \\ cos\psi & sin\psi \end{bmatrix} \begin{bmatrix} cos\phi sin\theta \\ sin\phi \end{bmatrix} = \begin{bmatrix} \frac{U_y}{U_1} \\ \frac{U_x}{U_1} \end{bmatrix} \tag{3.60}$$

Then multiply it with the inverse matrix

$$\begin{bmatrix} cos\phi sin\theta \\ sin\phi \end{bmatrix} = \begin{bmatrix} \frac{sin\psi U_y + cos\psi U_x}{U_1} \\ \frac{-cos\psi U_y + sin\psi U_x}{U_1} \end{bmatrix} \tag{3.61}$$

Finally the desired values for roll and pitch angles becomes

$$\begin{bmatrix} \phi_d \\ \theta_d \end{bmatrix} = \begin{bmatrix} sin^{-1}\left(\frac{-cos\psi U_y + sin\psi U_x}{U_1}\right) \\ sin^{-1}\left(\frac{sin\psi U_y + cos\psi U_x}{U_1 cos\phi}\right) \end{bmatrix} \tag{3.62}$$

3.6. Sliding mode controller: the general principle

Sliding mode controller (SMC) is a type of nonlinear control system methods that change the dynamics of the system by designing a multiple control structure to make sure that trajectories slide towards a switching surface. The control law works by switching from one continuous structure to another depending on position in state space. It is a robust control technique, which has the ability to compensate modeling errors, system's parameter differences and work for nonlinear and time varying systems. However, it has a draw back in forming a chattering effect that gives a high frequency oscillation [11, 12].

In designing a sliding mode control, there are two phases: choosing the sliding surface and forming a control law. Sliding surface is a surface that the system needs to slide and satisfy the design specifications. It is a geometrical locus that consists of boundaries. The control law is the second step that make system attractive to the desired state [13].

The general typical form of the sliding surface is expressed as [13]:

$$\mathrm{S(x)} = \left(\lambda_{\mathrm{x}} + \frac{\mathrm{d}}{\mathrm{dt}}\right)^{\mathrm{f}-1} \mathrm{e(x)} \tag{3.63}$$

where λ_{x} is the tuning parameter for defining the S(x) performance,

X is a control variable or state vector,

e(x) is a tracking error defined as desired value minus actual value and

f is the relative degree between input and output.

The second step of designing the control law also consists of two components: the linear $U_{eq}(t)$ and the nonlinear component $U_D(t)$ [1, 11].

$$U(t) = U_{eq}(t) + U_D(t) \tag{3.64}$$

The linear part protects the movement of the system on the sliding surface whenever the system is on the surface. This part of controller that maintains the sliding condition must satisfy a condition

$$\dot{S} = 0$$

The discontinuous part of the control law is used to compensate any variation of the state trajectories from the sliding surface in order to reach it. And can be expressed as

$$U_D(t) = K\ sign(s(t)) \tag{3.65}$$

where $$sign\big(s(t)\big) = \begin{cases} -1 & if\ s < 0 \\ 0 & if\ s = 0 \\ 1 & if\ s > 0 \end{cases}$$

and K is a design constant that must be greater than zero to satisfy the Lyapunov stability condition $s\dot{s} > 0$.

Sliding mode controller principle is shown in Figure 3.7 by following the sliding surface. When the sliding surface is cached by the system, then it will determine the way of system in a closed loop.

Figure 3.7: Graphical demonstration of SMC

3.7. SMC in Hexacopter

In this work, six sliding mode controllers are used to control all the states of the Hexacopter dynamics.

3.7.1. Altitude controller

The first step in controlling all the states of the Hexacopter is designing the sliding surface. Since it is trajectory tracking, it depends on the error.

$$s_z = \dot{e}_1 + \lambda_z e_1 \tag{3.66}$$

where λ_z is a tuning parameter

The error equation for the altitude is given as

$$e_1 = z_d - z \tag{3.67}$$

Where z_d is the desired state and z is the actual state

Inserting the error equation to the sliding surface gives

$$s_z = (\dot{z}_d - \dot{z}) + \lambda_z(z_d - z) \tag{3.68}$$

The derivative of the sliding surface defined in equation (3.66) by substituting equation (3.67) it gives

$$\dot{s}_z = \ddot{e}_1 + \lambda_z \dot{e}_1 = (\ddot{z}_d - \ddot{z}) + \lambda_z(\dot{z}_d - \dot{z}) \quad (3.69)$$

Substituting the altitude dynamics of the Hexacopter from equation (2.46) to the derivative of sliding surface gives

$$\dot{s}_z = \ddot{z}_d - (\frac{1}{m}(c\phi c\theta)U_1 - g) + \lambda_z(\dot{z}_d - \dot{z}) \quad (3.70)$$

where $c\phi$ is $cos\phi$ and $c\theta$ is $\cos\theta$

The next step is designing the sliding control law for the altitude that make the trajectories to attract to the surface and keep on sliding on it for all time.

$$U(t) = U_{eq}(t) + U_D(t) \quad (3.71)$$

Since the system is in sliding condition $U_{eq} = U_1$ and as $\dot{s}_z$=0, equation (3.70) gives the equivalent equation

$$U_{eq} = \frac{\mathrm{m}}{c\phi c\theta}[\ddot{z}_d + g + \lambda_z \dot{e}_1] \quad (3.72)$$

Then to design the discontinuous controller, the positive definite Lyapunov function must be defined to be

$$V = \frac{1}{2}S^2 > 0 \quad (3.73)$$

Moreover, the derivative of Lyapunov function at equation (3.73) which became $\dot{V} = S\dot{S} < 0$ must be negative definite. So for all $t > 0$ and $\mathrm{K_z} > 0$, the discontinuous control is formulated as

$$U_D = \mathrm{K_z sgn(s_z)} \quad (3.74)$$

Finally, the Altitude controller input becomes

$$U_1 = \frac{\mathrm{m}}{c\phi c\theta}[\ddot{z}_d + g + \lambda_z \dot{e}_1 + K_z sgn(s_z)] \quad (3.75)$$

3.7.2. Roll controller

Controlling a roll movement is the same procedure as altitude control in that choosing the sliding surface gives

$$s_\phi = \dot{e}_2 + \lambda_\phi e_2 \quad (3.76)$$

where $e_2 = \phi_d - \phi$

Derivating the sliding surface equation (3.76) and substituting the error equation yields

$$\dot{s}_\phi = \ddot{e}_2 + \lambda_\phi \dot{e}_2 = \ddot{\phi}_d - \ddot{\phi} + \lambda_\phi(\dot{\phi}_d - \dot{\phi}) \tag{3.77}$$

Substituting the dynamics equation of Hexacopter roll dynamics in equation (3.50) to the sliding surface equation (3.77), we obtain

$$\dot{s}_\phi = \ddot{\phi}_d - \dot{\psi}\dot{\theta}\left(\frac{J_{yy}-J_{zz}}{J_{xx}}\right) - \frac{J_r\Omega_r\dot{\theta}}{J_{xx}} - \frac{l}{J_{xx}}U_2 + \lambda_\phi(\dot{\phi}_d - \dot{\phi}) \tag{3.78}$$

To get the equivalent control equation, $\dot{s}_\phi = 0$ gives

$$U_{eq} = \frac{J_{xx}}{l}\left[\ddot{\phi}_d - \dot{\psi}\dot{\theta}\left(\frac{J_{yy}-J_{zz}}{J_{xx}}\right) - \frac{J_r\Omega_r\dot{\theta}}{J_{xx}} + \lambda_\phi(\dot{\phi}_d - \dot{\phi})\right] \tag{3.79}$$

The discontinuous part of controller selected for all $t > 0$, $K_\phi > 0$ becomes

$$U_D = K_\phi \operatorname{sgn}(s_\phi) \tag{3.80}$$

Finally, the roll controller that makes the system to the selected sliding surface by adding the linear and discontinuous components derived above, it gives

$$U_2 = \frac{J_{xx}}{l}\left[\ddot{\phi}_d - \dot{\psi}\dot{\theta}\left(\frac{J_{yy}-J_{zz}}{J_{xx}}\right) - \frac{J_r\Omega_r\dot{\theta}}{J_{xx}} + \lambda_\phi(\dot{\phi}_d - \dot{\phi}) + K_\phi \operatorname{sgn}(s_\phi)\right] \tag{3.81}$$

3.7.3. Pitch controller

Controlling the pitch dynamics has also the same procedure with the others designed before

The sliding surface will be

$$s_\theta = \dot{e}_3 + \lambda_\theta e_3 \tag{3.82}$$

where error $e_3 = \theta_d - \theta$

Derivating the sliding surface equation give

$$\dot{s}_\theta = \lambda_\theta \dot{e}_3 + \ddot{e}_3 = \ddot{\Theta}_d - \ddot{\Theta} + \lambda_\theta(\dot{\Theta}_d - \dot{\Theta}) \tag{3.83}$$

Inserting the pitch dynamics $\ddot{\theta} = \dot{\psi}\dot{\phi}\left(\frac{J_{zz}-J_{xx}}{J_{yy}}\right) - \frac{J_r\Omega_r\dot{\phi}}{J_{yy}} + \frac{l}{J_{yy}}U_3$ into the above equation gives

$$\dot{s}_\theta = \ddot{\theta}_d - \dot{\psi}\dot{\phi}\left(\frac{J_{zz}-J_{xx}}{J_{yy}}\right) + \frac{J_r\Omega_r\dot{\phi}}{J_{yy}} - \frac{l}{J_{yy}}U_3 + \lambda_\theta(\dot{\theta}_d - \dot{\theta}) \tag{3.84}$$

Next the control law is designed and to find the equivalent equation, we set $\dot{s}_\theta = 0$

Then

$$U_{eq} = \frac{J_{yy}}{l}\left[\ddot{\theta}_d - \dot{\psi}\dot{\phi}\left(\frac{J_{zz}-J_{xx}}{J_{yy}}\right) + \frac{J_r\Omega_r\dot{\phi}}{J_{yy}} + \lambda_\theta(\dot{\theta}_d - \dot{\theta})\right] \tag{3.85}$$

The discontinuous part of the pitch dynamics for all $t > 0, K_\theta > 0$ becomes

$$U_D = K_\theta \mathrm{sgn}(s_\theta) \tag{3.86}$$

Finally, the pitch controller becomes

$$U_3 = \frac{J_{yy}}{l}\left[\ddot{\theta}_d - \dot{\psi}\dot{\phi}\left(\frac{J_{zz}-J_{xx}}{J_{yy}}\right) + \frac{J_r\Omega_r\dot{\phi}}{J_{yy}} + \lambda_\theta(\dot{\theta}_d - \dot{\theta}) + K_\theta sgn(s_\theta)\right] \tag{3.87}$$

3.7.4. Yaw controller

As using the same procedure with other controllers, first design a sliding surface

$$s_\psi = \dot{e}_4 + \lambda_\psi e_4 \tag{3.88}$$

where error $e_4 = \psi_d - \psi$

The derivative of the sliding surface give

$$\dot{s}_\psi = \ddot{e}_4 + \lambda_\psi \dot{e}_4 == \ddot{\psi}_d - \ddot{\psi} + \lambda_\psi(\dot{\psi}_d - \dot{\psi}) \tag{3.89}$$

Inserting the yaw dynamics equation $\ddot{\psi} = \dot{\phi}\dot{\theta}[\frac{Jxx-Jyy}{Jzz}] + \frac{1}{Jzz}U_4$ to the above equation (3.89), it gives

$$\dot{s}_\psi = \ddot{\psi}_d - \dot{\phi}\dot{\theta}\left[\frac{Jxx-Jyy}{Jzz}\right] - \frac{1}{Jzz}U_4 + \lambda_\psi(\dot{\psi}_d - \dot{\psi}) \tag{3.90}$$

Next designing the yaw dynamics controller the equivalent control becomes

$$U_{eq} = J_{zz}\left[\ddot{\psi}_d - \dot{\phi}\dot{\theta}[\frac{Jxx-Jyy}{Jzz}] + \lambda_\psi(\dot{\psi}_d - \dot{\psi})\right] \tag{3.91}$$

The discontinuous component chosen for all $t > 0, K_\psi > 0$

$$U_D = K_\psi \mathrm{sgn}(s_\psi) \tag{3.92}$$

Finally, adding the continuous and discontinuous components of the controller, the yaw dynamics controller becomes

$$\mathrm{U}_4 = J_{zz}\left[\ddot{\psi}_d - \dot{\phi}\dot{\theta}[\frac{Jxx-Jyy}{Jzz}] + \lambda_\psi(\dot{\psi}_\mathrm{d} - \dot{\psi}) + K_3 sgn(s_3)\right] \tag{3.93}$$

3.7.5. Linear X motion controller

Linear x motion is the movement through the x direction in which it gives the desired value for the x movement.

Choosing the sliding surface for the movement

$$s_x = \dot{e}_x + \lambda_x e_x \tag{3.94}$$

where $e_x = x_d - x$

The time derivative becomes

$$\dot{s}_x = \ddot{e}_x + \lambda_x \dot{e}_x \tag{3.95}$$

Substituting the error equation to time derivate of surface equation give

$$\dot{s}_x = \ddot{e}_x + \lambda_x \dot{e}_x = \ddot{x}_d - \ddot{x} + \lambda_x(\dot{x}_d - \dot{x}) \tag{3.96}$$

From the dynamic model of the Hexacopter translational dynamics for position x in equation (3.44) and inserting to the above equation gives

$$\dot{s}_x = \ddot{x}_d - (\frac{1}{m}[cos\phi cos\psi sin\theta + sin\psi sin\phi]U_1) + \lambda_x(\dot{x}_d - \dot{x}) \tag{3.97}$$

Substituting U_x by $cos\phi cos\psi sin\theta + sin\phi sin\psi$ yields

$$\dot{s}_x = \ddot{x}_d - \frac{1}{m}U_x U_1 + \lambda_x(\dot{x}_d - \dot{x}) \tag{3.98}$$

Finding the value for equivalent equation by making $\dot{s}_x = 0$ it becomes

$$U_x = \frac{m}{U_1}[\ddot{x}_d + \lambda_x(\dot{x}_d - \dot{x})] \tag{3.99}$$

Then the discontinuous control part that makes the Lyapunov function V positive definite is designed as

$$U_D = K_x sign(S_x) \tag{3.100}$$

Then the final controller for linear x position is expressed as

$$U_x = \frac{m}{U_1}[\ddot{x}_d + \lambda_x(\dot{x}_d - \dot{x})] + K_x sign(S_x) \qquad (3.101)$$

3.7.6. Linear y motion controller

The linear y motion is designed by applying similar procedure with the above mentioned controllers

First choosing the sliding surface for the movement

$$s_y = \dot{e}_y + \lambda_y e_y \qquad (3.102)$$

where the error equation $e_y = y_d - y$

derivating the sliding surface and substituting the error equation on it gives

$$\dot{s}_y = \ddot{e}_y + \lambda_y \dot{e}_y = \ddot{y}_d - \ddot{y} + \lambda_y(\dot{y}_d - \dot{y}) \qquad (3.103)$$

From the dynamic model of the Hexacopter translational dynamics for position y in equation (3.45) and inserting to the above equation gives

$$\dot{s}_y = \ddot{y}_d - (\frac{1}{m}[cos\phi sin\theta sin\psi - sin\phi cos\psi]U_1) + \lambda_y(\dot{y}_d - \dot{y}) \qquad (3.104)$$

Substituting U_y in place of $cos\phi sin\theta sin\psi - sin\phi cos\psi$ we get

$$\dot{s}_y = \ddot{y}_d - \frac{1}{m}U_y U_1 + \lambda_y(\dot{y}_d - \dot{y}) \qquad (3.105)$$

Then finding the equivalent equation at $\dot{s}_y = 0$ yields

$$U_y = \frac{m}{U_1}[\ddot{y}_d + \lambda_y(\dot{y}_d - \dot{y})] \qquad (3.106)$$

The discontinuous control component at $K_x > 0$ becomes

$$U_D = K_x sign(S_x) \qquad (3.107)$$

The final controller for the linear y position controller is

$$U_y = \frac{m}{U_1}[\ddot{y}_d + \lambda_y(\dot{y}_d - \dot{y}) + K_x sign(S_x)] \qquad (3.108)$$

Chapter Four

Simulation results and analysis

This chapter provides simulation based evidence on the trajectory tracking performance of applied controller for the designed Hexacopter. The design is implementing into a software environment specifically on MATLAB Simulink. This chapter also presents discussion of the simulation results.

4.1. Desired trajectory

The desired trajectory for the flight of the Hexacopter is designed as

$$X_d = \sin(t) + \cos(t)$$

$$Y_d = \cos(t)^2$$

$$Z_d = \cos(t) + t$$

$$\psi_d = \cos(2 * t) + \sin(t)$$

4.2. Closed loop simulations

The gain parameters λ_i and K_i for the altitude, attitude and position states of a Hexacopter are designed by using a trial and error method that would give a least possible settling time and minimize the error. Table 3 shows the values for the parameters got by try and error.

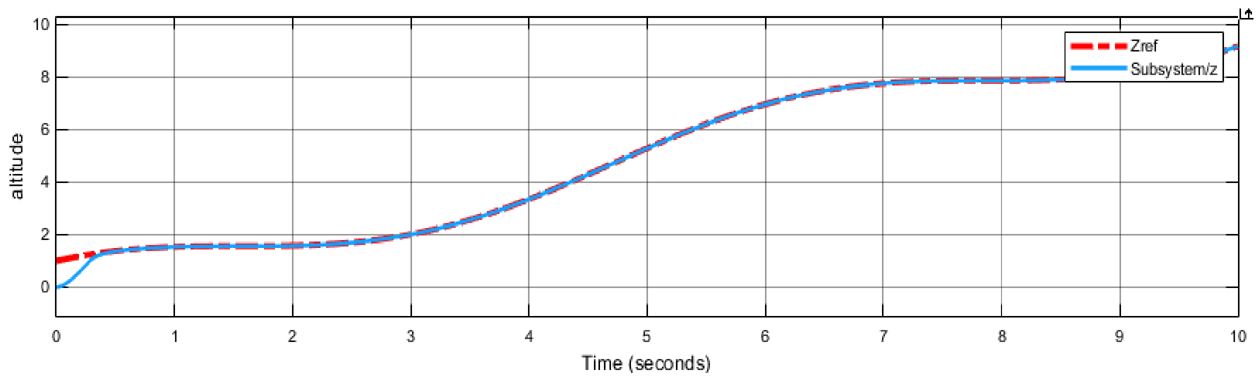
Table 3: Numerical values for constant λ_i and K_i

Dynamics	λ_i	K_i	Settling time (seconds)
X position	15	30	0.65
Y position	15	50	0.5
Z altitude	15	100	0.4
Yaw (ψ)	20	100	0.3

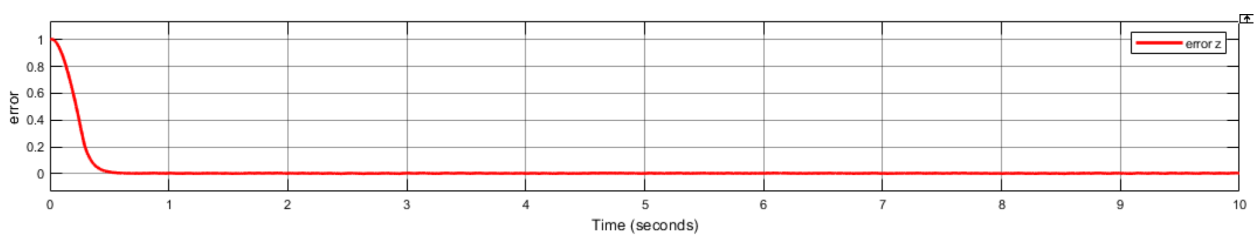
4.2.1. Z Altitude tracking

Notice that the SMC controller input U1 was designed for the altitude control in the previous chapter, and applying it to the system dynamics for tracking the given reference value the response will looks like in Figure (4.1a). Figure (4.1) shows the tracking ability of the Hexacopter for a given reference altitude and the respective error between the desired and actual values.

It is clear to see from the simulation that SMC has a good tracking response for a given desired altitude. The controller follows the given desired trajectory with in a small response time. In addition, the steady state error become zero within a few seconds. Tuning the value of parameters λ_z and K_z was made by trial and error tuning method until getting a better response while following desired value as demonstrated in Table 3 .



(a)

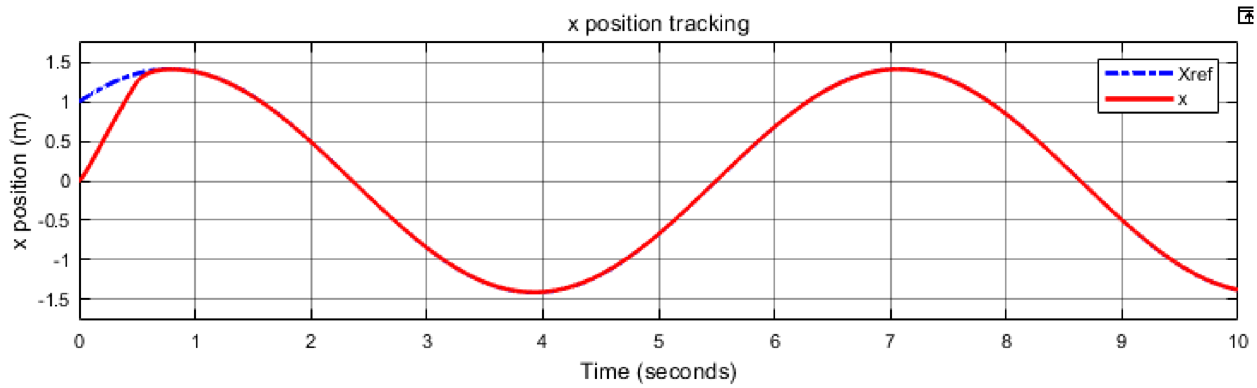


(b)

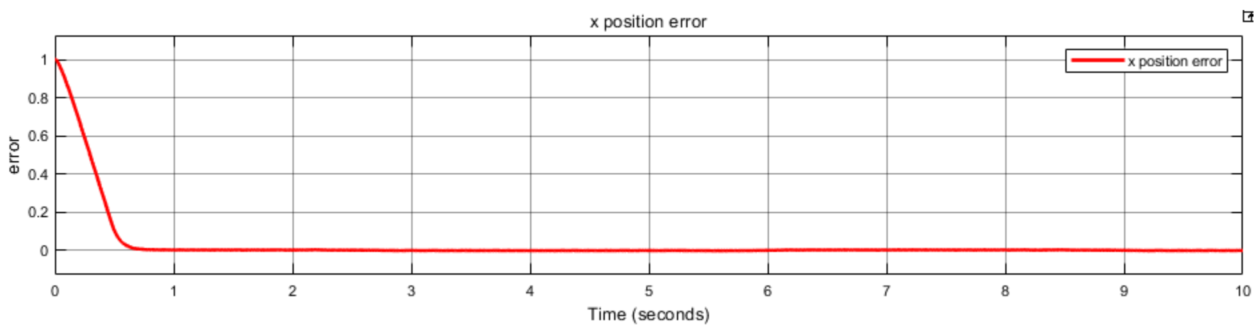
Figure 4.1: a) reference tracking and (b) tracking error responses for the altitude

4.2.2. X position tracking

The result depicted in Figure (4.2) presents the simulation response of the position tracking in x-axis by using sliding mode controller (SMC) and the error diagram between the desired and actual values. The result shows that within a short period of time the controller gets the desired value and give better response. It also presents that the error value becomes approximately zero with in a finite time.



(a)

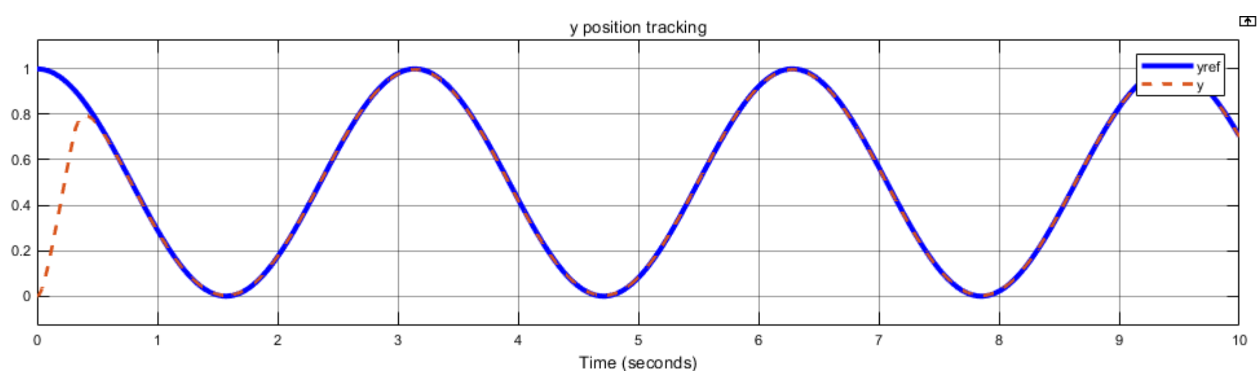


(b)

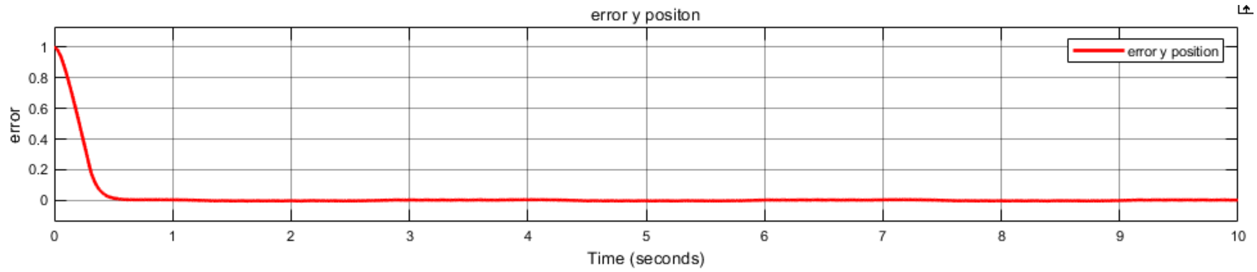
Figure 4.2: (a) trajectory tracking and (b) tracking error response of the x position simulation

4.2.3. Y position tracking

Figure (4.3) describes the simulation response of y position sliding mode controller for following of a given trajectory. It can be seen from the figure that the controller gets the reference in approximately 0.5seconds, which is minor, and the error becomes zero after this second.



(a)

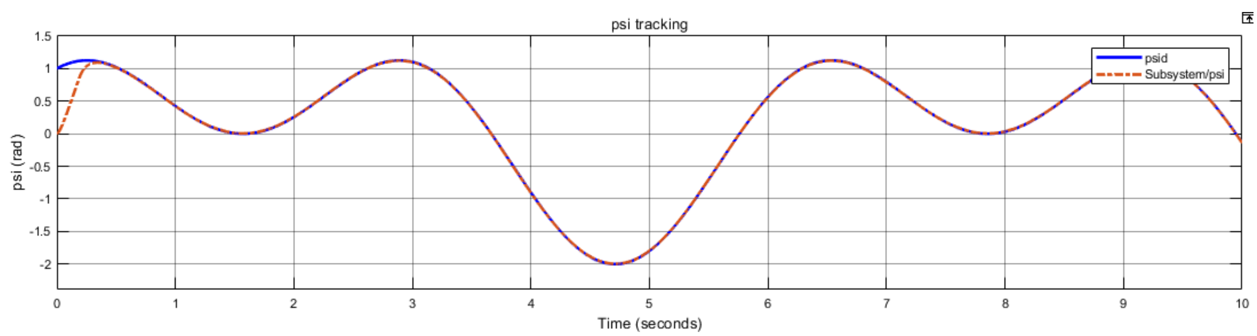


(b)

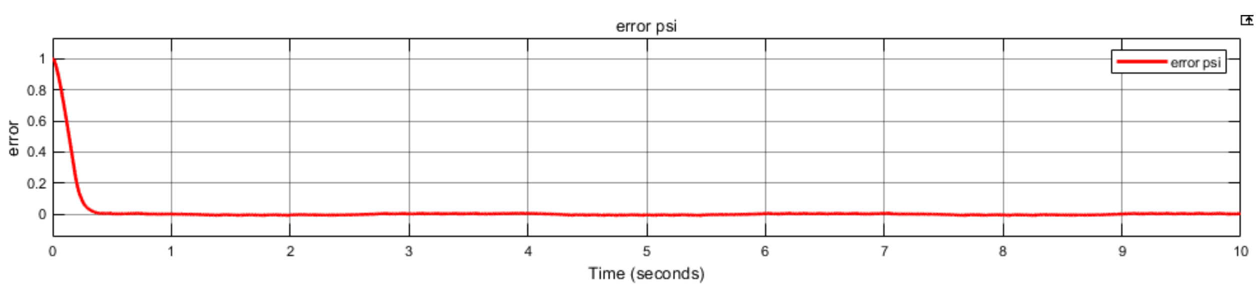
Figure 4.3: (a) trajectory tracking (b) error response of y position

4.2.4. Psi tracking

The tracking response of psi in Figure 4.4 shows that the controller takes a few seconds time to get a given desired value and have zero error thereafter.



(a)

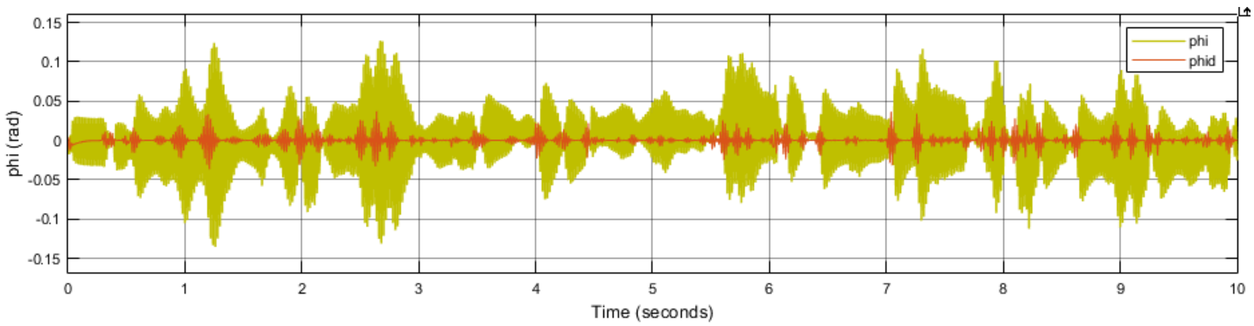


(b)

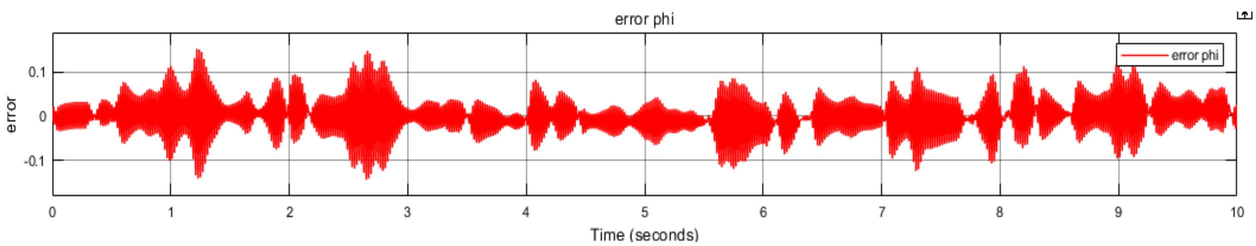
Figure 4.4: (a) output tracking response of psi (b) error of the response of psi

4.2.5. Attitude phi and theta tracking

Figure (4.5) and (4.6) shows the simulation response of the rotational components phi and theta in following a given trajectory and their respective tracking errors. As seen from the figure, the response is highly oscillatory and it follows the desired input with a minimum error. However, it is not exact tracking. This is because of the gain parameters and the controller.

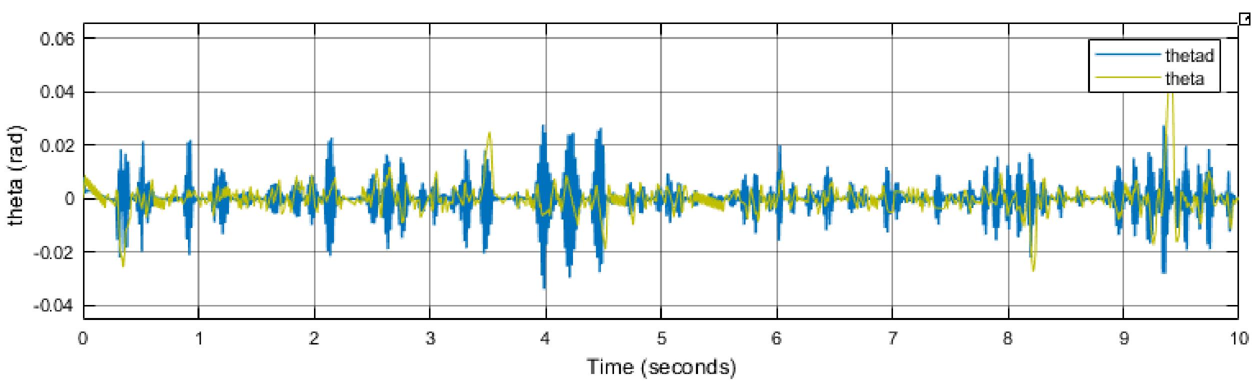


(a)

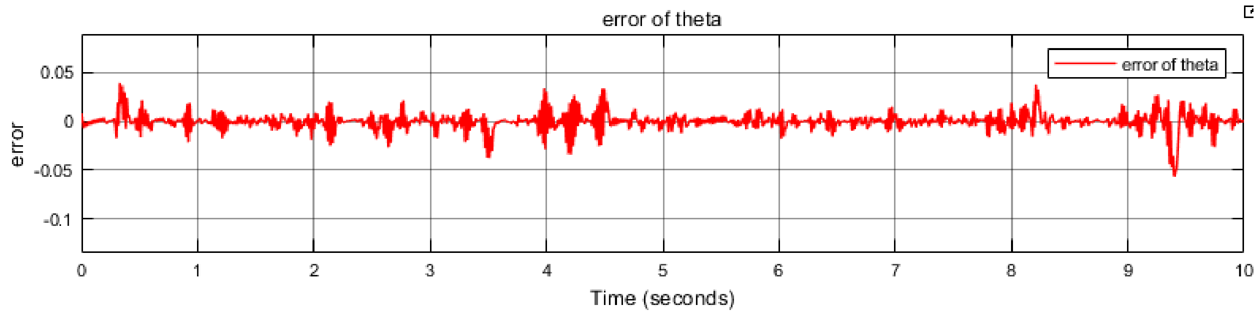


(b)

Figure 4.5: (a) tracking response of phi (b) tracking error of phi



(a)

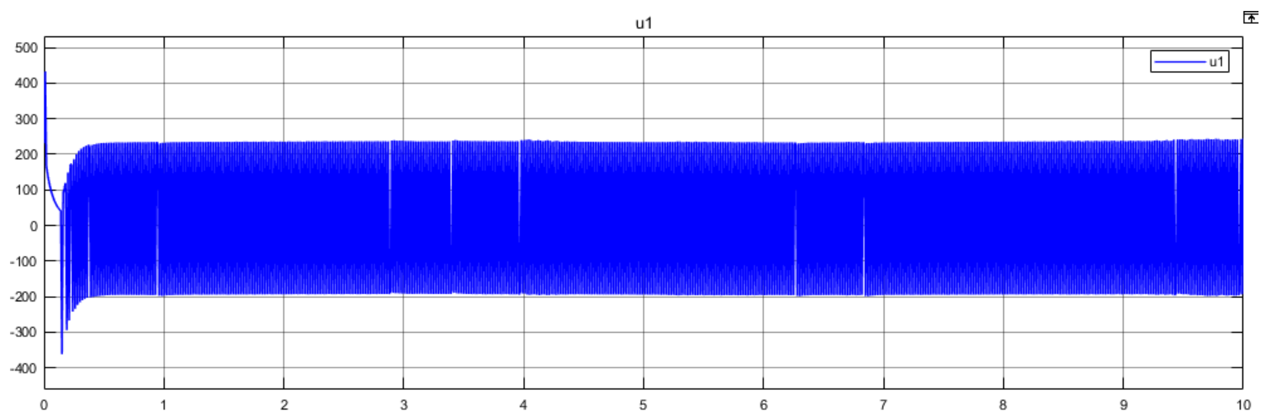


(b)

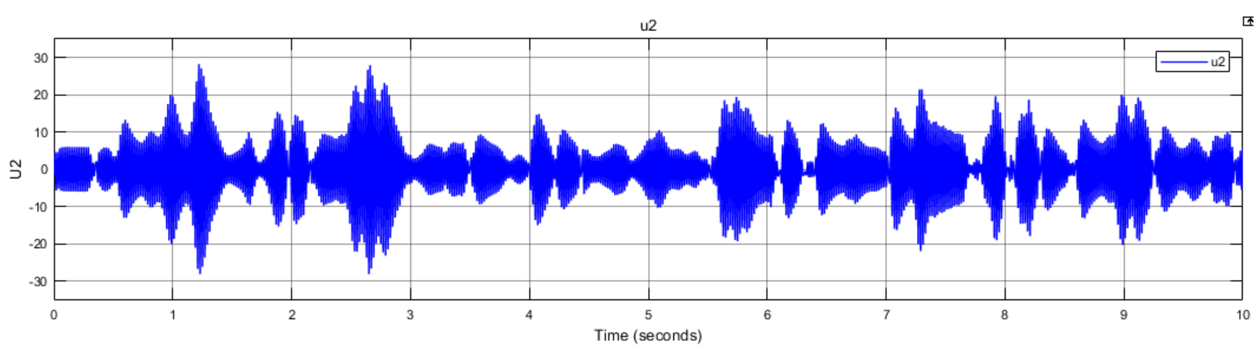
Figure 4.6: (a) tracking response of theta (b) tracking error of theta

4.2.6. Control inputs response

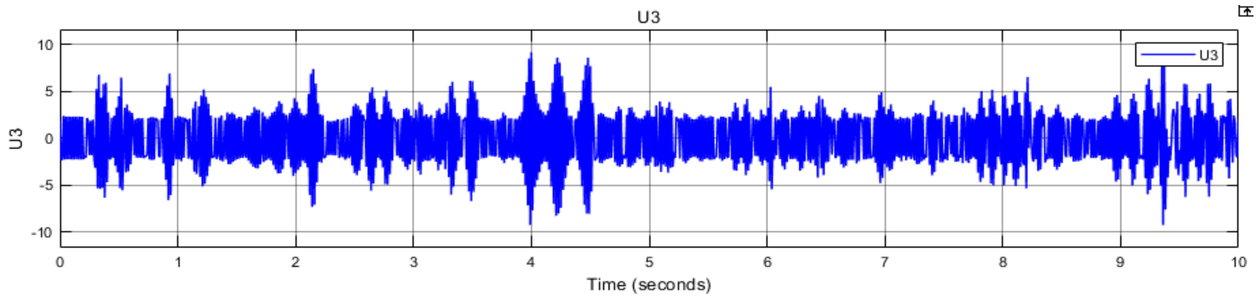
The response for the control inputs designed for SMC controller is shown in Figure 4.7.



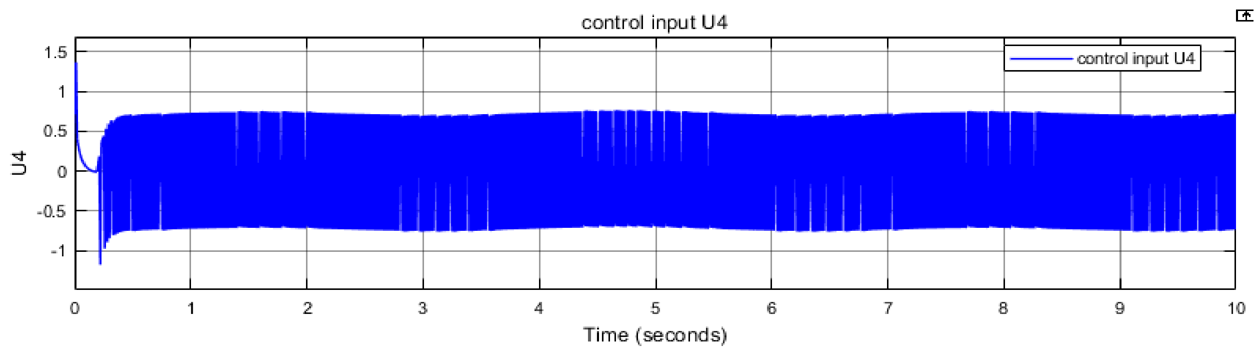
(a) Control input U1



(b) Control input U2



(c) Control input U3



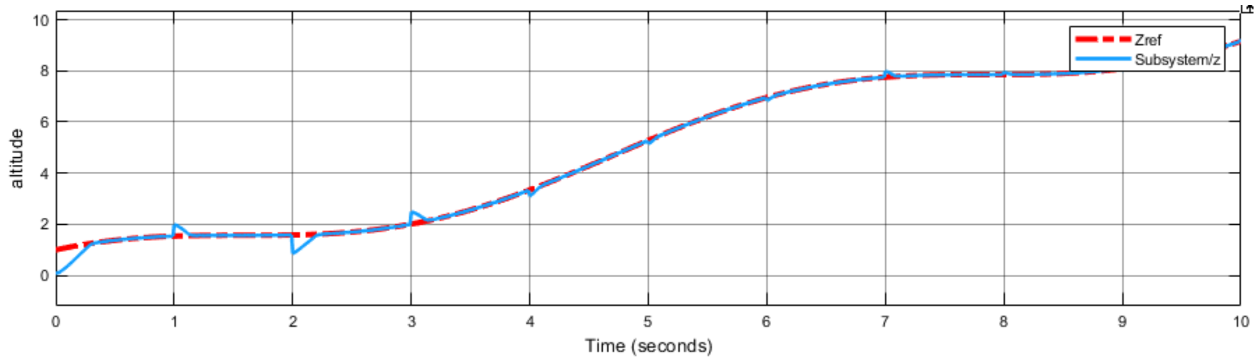
(d) Control input U4

Figure 4.7: SMC control inputs simulation response

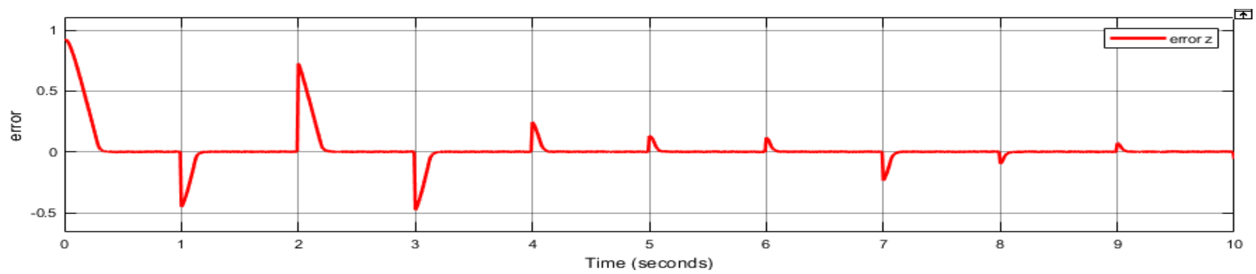
4.2.7. Tracking response with disturbance

Since Hexacopter have lightweight, low power and slow speed, they can easily influenced by external disturbances like wind, obstacles. This may disturb the flight performance and stability of system. To overcome this problem, the system must be controlled by adding these unknown disturbances. Figures 4.8- 4.11 shows the tracking response for the parameters when some random disturbance is added to the system. The response shows that when some disturbance is added to the system the controller gets the desired value with short time with minimum error values.

4.2.7.1. Z altitude position tracking



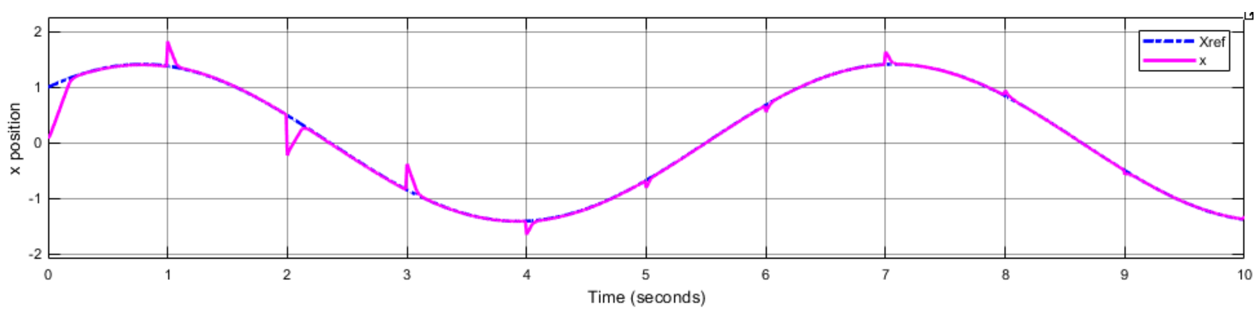
(a)



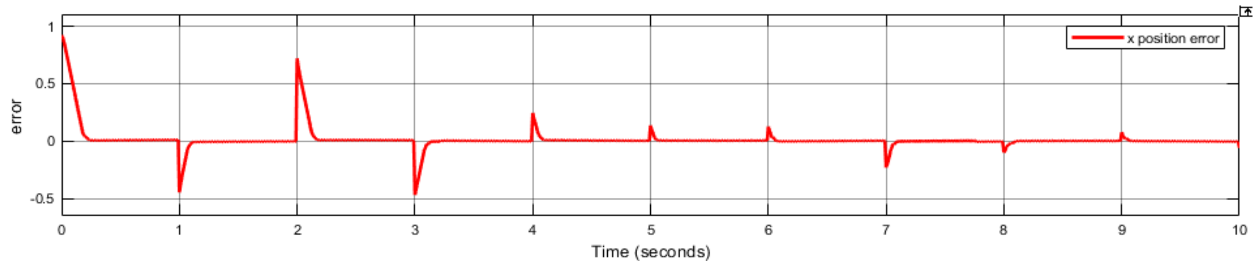
(b)

Figure 4.8: (a) z tracking response with disturbance (b) tracking error

4.2.7.2. X position tracking



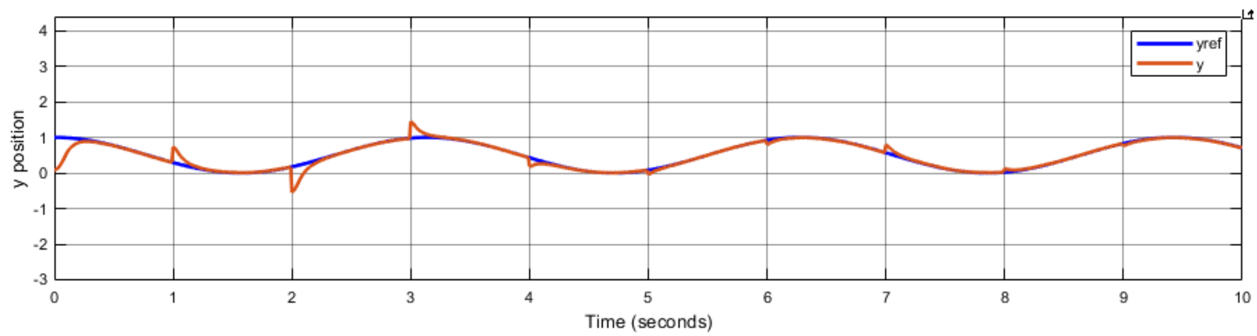
(a)



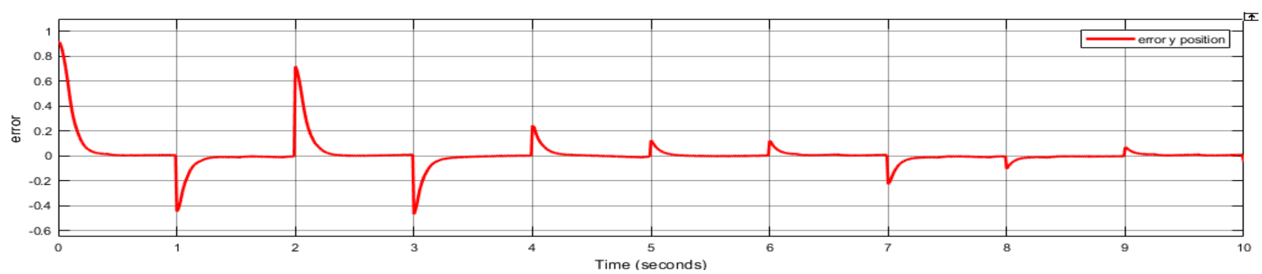
(b)

Figure 4.9: (a) x position tracking response with disturbance (b) tracking error

4.2.7.3. Y position tracking



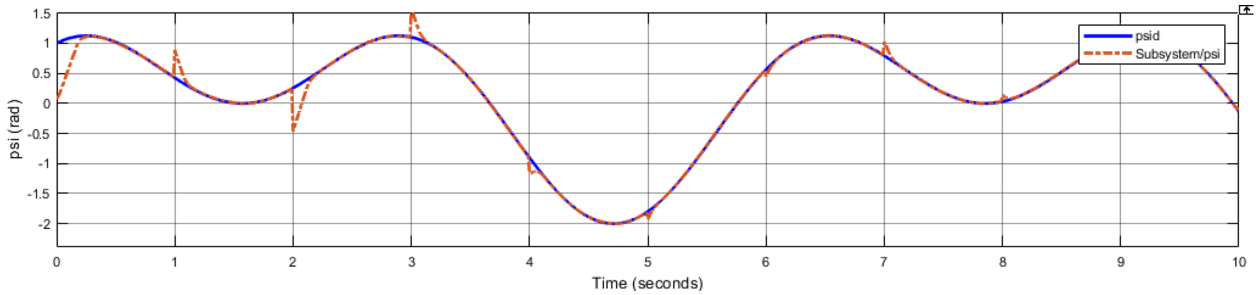
(a)



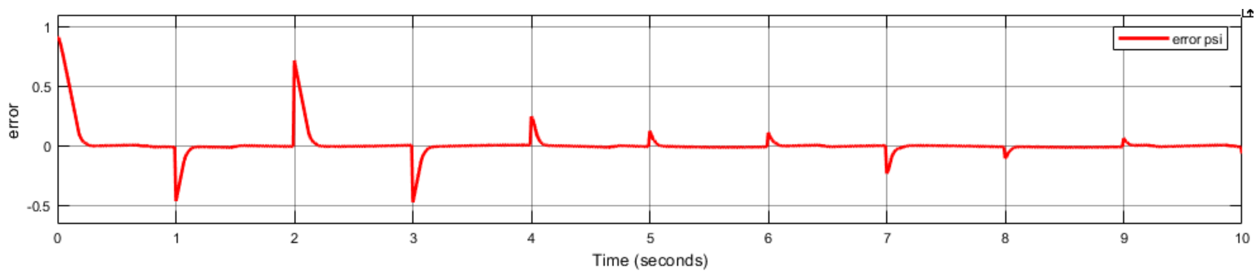
(b)

Figure 4.10: (a) y position tracking response with disturbance (b) tracking error

4.2.7.4. Psi tracking



(a)



(b)

Figure 4.11: (a) psi tracking response with disturbance (b) tracking error

Finally, the applied SMC controller for both rotational and translational components of a Hexacopter gives a better response in following the given reference value with in a small response time and steady state error. In addition, when random disturbance is added to the system it gives a good tracking response.

Chapter Five

Conclusions and Recommendations

5.1. Conclusions

The main objective of this thesis work was to drive a mathematical model for a multirotor called Hexacopter and to design a nonlinear controller that makes the system to track a specified reference value for all the states of Hexacopter such as position, altitude, attitude and heading and to justify the performance of the controller.

The mathematical model of a Hexacopter was formulated by using newton Euler formalism by assuming the Hexacopter as a rigid body. A robust and nonlinear sliding mode controller (SMC) was then developed. The entire system of the designed mathematical model of the Hexacopter was implemented on MATLAB Simulink depending on the designed mathematical model of the Hexacopter. The simulation environment was used to evaluate the correctness of the derived model and the performance of the applied controller under a given condition.

Simulation results have shown that sliding mode controller (SMC) performs good in tracking a reference value within a short settling time approximately in 0.5 seconds and has almost zero error value. It has a significance on robustness and stability response.

5.2. Recommendations

The controller applied to the system has a good performance for tracking a reference but the tuning method applied for the parameters of SMC is by trial and error. Developing a tuning equation to better simplify the system is left for future study. Moreover, the control design for rotational components could be improved by using a mixture of controllers such as adding fuzzy or neural network with SMC to make the system more intelligent.

The dynamic model of a Hexacopter describes the dynamics of a real Hexacopter. A better physical model of the system for instance, by adding a disturbance effect would make the system more suitable for designing perfect controller and make the simulation design more realistic.

References

[1] Heba talla Mohamed, "Dynamic modeling and control of a Quadrotor using linear and nonlinear approaches," MSc thesis, American university, school of science and engineering, Cairo, 2014.

[2] Luis Rodolfo G.C, Alejandro Enrique D.L, Rogelio Lozano, Claude Pegard, *Advances in industrial control, Quad rotorcraft control*, 2012.

[3] Ruth Tesfaye, "Modeling and control of quad rotor unmanned aerial vehicle at hovering position," MSc thesis, Addis Ababa University, ECE department, Addis Ababa, December 2012.

[4] Johan Fogleberg, "Navigation and autonomous control of Hexacopter in indoor environments," MSc thesis, Lund University, Sweden, 2013.

[5] M.Moussid, A.Sayouti, H.Medromi, "Dynamic modelling and control of a Hexarotor using linear and nonlinear methods," *International Journal of applied information systems*-ISSN: 2249-0868, vol. 9, no. 5, pp. 9-17, August 2015.

[6] A.Alaimo, V.Artale, C.L.R.Millazo, A.Ricciardello, "PID controller applied to Hexacopter flight," *Journal of intelligent and robot system*, vol. 73, pp.261-270, October 2013.

[7] A.Alaimo, V.Artale, C.L.R.Millazo, C.Orlando, A.Ricciardello, "LQR-PID control applied to Hexacopter flight," *European society of computational methods in science and engineering*, vol. 9-10, no. 3-4, pp. 47-56, 2016.

[8] Geovanny D.C.G, O.Camacho, Control of Hexacopter: A sliding mode control and PID controller, *Rev. Tec. Ing. Universidad Del Zulia*, vol. 39, no. 3, pp. 137-144, Venezuela, December 2016.

[9] Tobias Magnusson, "Attitude control of a Hexarotor," MSc thesis, Linköpings universitet, EE department, Sweden, 2014.

[10] Farhad Parivash, Ali Ghasemi, Trajectory tracking control for a quadroter using fuzzy PID control scheme, *IEEE Fourth Int. conf. on KBEI*, pp. 553-558, Iran, Dec.22, 2017.

[11] Andrzej Bartoszewicz, *Sliding mode control*, 1st ed. India: intech publisher, March 2011.

[12] Pablo Proano, Linda Capito, Andres Rosales, Oscar Camacho, Sliding Mode Control: Implementation like PID for trajectory-tracking for mobile robots, *IEEE Asia-pacific conf. on CASE*, pp. 220-225, Asia, 2015.

[13] Piazza d’armi, “A Quick introduction to sliding mode control and its applications,” http://www.diee.unica.it.html.

[14] S.Lindblom, A.Lundmark, “Modeling and control of a Hexarotor UAV,” MSc thesis, Linköpings universitet, EE department, Sweden, 2015.

[15] Matteo Vanin, “Modeling, identification and navigation of autonomous air vehicles,” MSc thesis, university of padova, Information engineering department, 2012-2013.

[16] Tommaso Bresciani. Modeling, identification and control of a quadcopter helicopter, Lund University, October 2008.

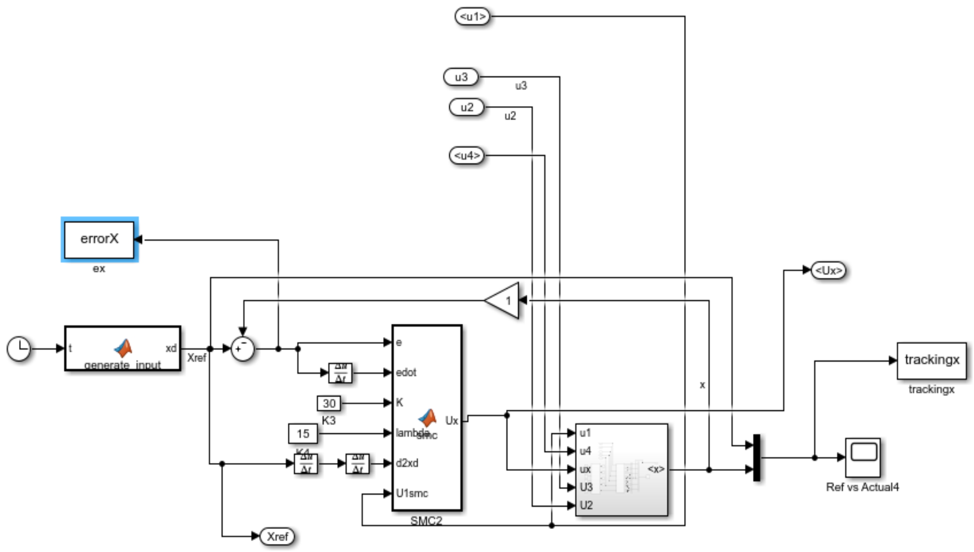
[17] V.Artale, C.Milazzo, A.Ricciardello. Mathematical modeling of Hexacopter, *Applied mathematical sciences*, vol. 7, no. 97, pp. 4805-4811, Italy 2013.

[18] Murray L. Ireland, A.Vargas, D.Anderson. A comparison of closed loop performance of multirotor configurations using nonlinear dynamic inversion control, *Aerospace*, vol. 2, pp.325-352, June 2015.

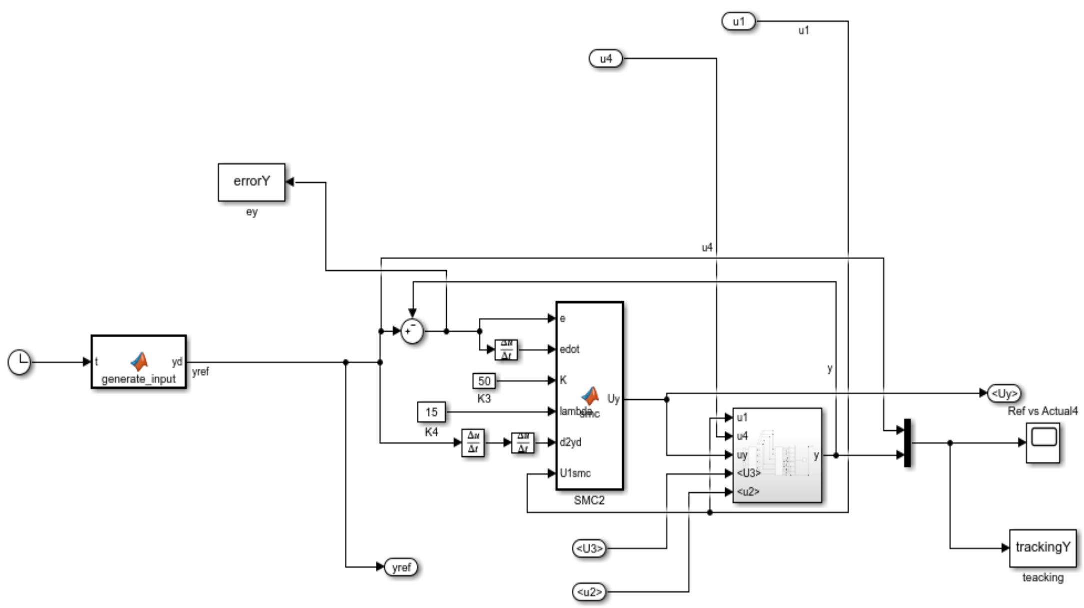
Appendix

Simulink Block diagram

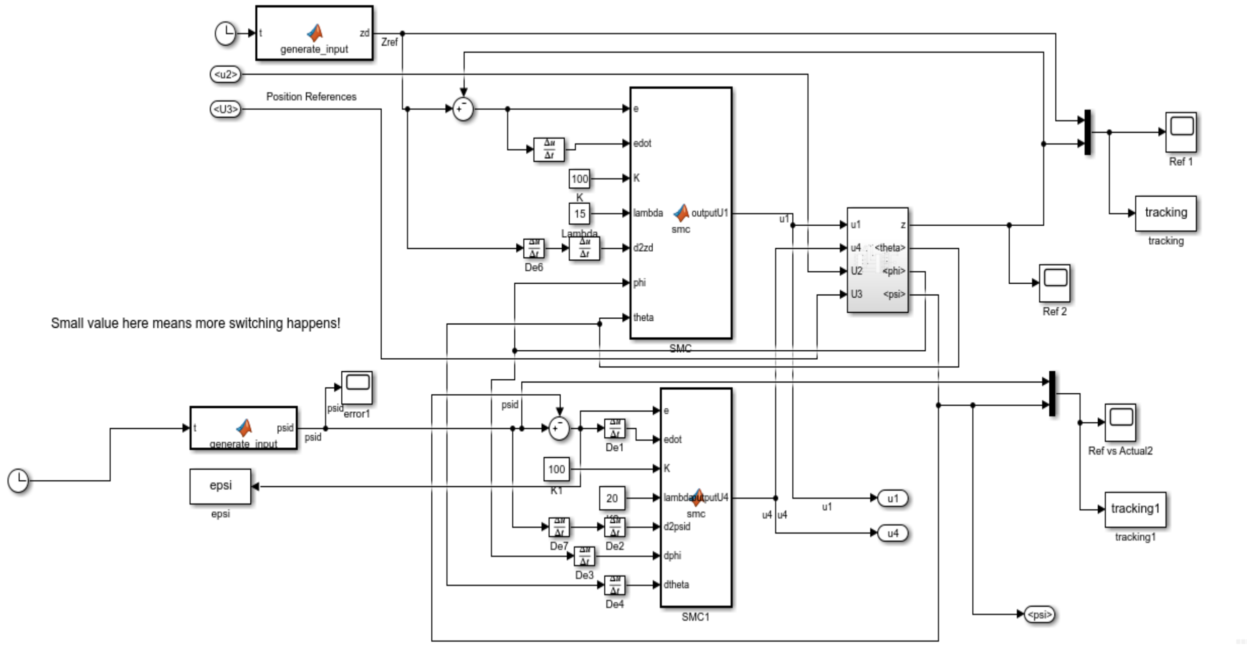
A. for x position tracking



B. for y position tracking



C. altitude and psi tracking



D. theta and phi tracking